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## Topics in electroweak baryogenesis: the sphaleron and t-violation

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**Topics in electroweak baryogenesis: The sphaleron and  
T-violation**

**Lee, SeungKoog, Ph.D.**

**Iowa State University, 1993**

**U·M·I**

**300 N. Zeeb Rd.  
Ann Arbor, MI 48106**



**Topics in electroweak baryogenesis: The sphaleron and t-violation**

**by**

**SeungKoog Lee**

**A Dissertation Submitted to the  
Graduate Faculty in Partial Fulfillment of the  
Requirements for the Degree of  
DOCTOR OF PHILOSOPHY**

**Department: Physics and Astronomy  
Major: High Energy Physics**

**Approved:**

Signature was redacted for privacy.

**In Charge of Major Work**

Signature was redacted for privacy.

**For the Major Department**

Signature was redacted for privacy.

**For the Graduate College**

**Iowa State University  
Ames, Iowa**

**1993**

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## GENERAL INTRODUCTION

On the observational basis of the absence of anti matter, the possibility of the violation of baryon number and lepton number in nature has been speculated for many years. It was pointed out by Sakahrov (1967) that excess baryons maybe generated in the early stages of the universe if the interactions among elementary particles in the hot plasma after the big bang violate charge conjugation (C), the combination of charge conjugation and parity (CP), and baryon number (B). There are two ways to understand the baryon number violation from the theoretical point of view. The first is to study the non-perturbative effect at high energy or at high temperature regime where the violation of baryon number is mediated by the sphaleron configuration. The second is the perturbative phenomenon at the low energy region such as CP-violation in Kaon decay.

### Organization

In paper I, we investigated the sphaleron energy change due to the higher dimension operators (i.e., greater than 4) due to the beyond the standard model forces because the sphaleron energy is the most important factor in setting the rate of baryon generation. In Chapter 2, we analyze the mathematical structure of the finite energy static field. In Chapter 3, we construct the Non-Contractible Loop (NCL) in the configuration space and examine the SU(2)- and SU(2)XU(1)-sphaleron solutions. In Chapter 4, we investigate the effect of the dimension 6 operators on the sphaleron energy.

In paper II, we considered T-violation effect induced in the CPT theorem, since T-violation is equivalent to CP-violation. This calculation provides us a small window for detecting new CP-violation effect which is a necessary ingredient in the understanding of

baryon asymmetry. In Chapter 2, we will review briefly the discrete symmetry C, P, T and CPT theorem. In Chapter 3, a short discussion on the CP violation in the Kaon decays is given, and in Chapter 4, we analyze the radiative decay. In Chapter 5, we calculate the transverse muon polarization induced by the electromagnetic final state interaction in the radiative decay.

Finally, the general conclusions are presented at the end of this dissertation.

PAPER I

THE SPHALERON SOLUTION IN EFFECTIVE MODEL

## CHAPTER 1. INTRODUCTION

All experimental results for the electromagnetic and weak interactions agree at present with the so-called standard model, i.e., Glashaw-Weinberg-Salam model. The standard model belongs to the class of Yang-Mills-Higgs theories, which means that the gauge group is non-Abelian ( $SU(2) \times U(1)$ ) and that some of the gauge bosons acquire a mass by the Higgs mechanism. The content of the standard model is generally described in terms of its particles: the gauge bosons (massive  $W^\pm$ ,  $Z^0$  and a massless photon  $\gamma$ ), the fermions (quarks and leptons in special representations of the gauge group) and a Higgs scalar field. All these particles can be considered as small fluctuations about the vacuum. But due to the non-trivial structure of the vacuum of the non-Abelian gauge theory, it is widely believed that the standard electroweak theory has a static, but unstable, classical solution.

Only quite recently has it been realized that the standard model may exhibit important non-perturbative effects. Remarkably, this happens despite the fact that the theory does not have an instanton or soliton solution in it. The non-perturbative structure is characterized by a different kind of classical solutions that are inherently unstable. Such static, but unstable, finite energy solutions of the classical field equations are called "sphaleron" (the Greek adjective "sphaleros" means unstable, ready to fall). Geometrically, the sphaleron lies on the top of a non-contractible loop in the configuration space of the finite-energy solutions of the equations of motion, and is a saddle point of the space. It separates vacua of different winding numbers and is considered to be directly associated with the vacuum to vacuum transition which induces the baryon (lepton) number violation in the standard model. It also sets the energy scale at which the non-trivial structure of the configuration space (the mathematical space consisting of all finite energy, static field configurations) becomes apparent.

The energy of the sphaleron was first calculated with vanishing Weinberg mixing angle, i.e., the  $SU(2)$ -Higgs model. The sphaleron energy has also been calculated in several non-standard models, and the results showed that the modifications of the sphaleron energy are small. The energy density, the total energy, and the magnetic moment vary by at most a few percent in the various cases. Hence the sphaleron energy is a remarkably stable quantity against the model variation, and the calculation of baryon number violation based on the  $SU(2)$  - Higgs model remains valid.

In all the above-mentioned investigations, the models considered are extensions of the standard electroweak interactions. We will investigate the effect on the sphaleron energy and the classical solutions of the gauge and Higgs fields due to physics beyond the standard model which is represented by the presence of effective terms in the Lagrangian. We restrict ourselves to operators up to dimension 6. The sphaleron energy is calculated in both perturbative and non-perturbative approaches.

In Chapter 2, we analyze the mathematical structure of the finite energy static field. In Chapter 3, we construct the Non-Contractible Loop (NCL) in the configuration space and examine the  $SU(2)$ -and  $SU(2) \times U(1)$ -sphaleron solutions. In Chapter 4, we investigate the effect of the dimension 6 operators on the sphaleron energy, and in Chapter 5 we present our conclusions.

## CHAPTER 2. FINITE ENERGY SOLUTIONS IN FIELD THEORIES

### 2.1. Classification of the Configurations

A classical field is a continuous map from a (space-time) manifold  $M$  to a topological space  $T$ , the field space.

$$\phi: x \in M \rightarrow \vec{\phi}(x) \in T$$

For example,  $T$  is a two sphere  $S^2$  for the  $O(3)$  model where all fields satisfy the constraint  $|\vec{\phi}(x)|^2 = 1$ . In other words, a classical field can be viewed as a family of maps from a space manifold  $M_0$  to the field space indexed by the real variable time. We will consider only the case where  $M$  is a flat (i.e., Minkowski or Euclidean) manifold. In other words, we ignore the gravitational effects by confining our attention to the flat manifold. Typically  $M = R^{d+1}$  for some integer  $d$  (space dimension).  $T$  will be assumed to be a compact topological space, a quotient space, or a compact Lie group, etc. [1]

We will study the smooth field configurations and their topological properties in the static case. With the dynamics being governed by an action principle, i.e., by evolution equations obtained by applying the variation principle to the action  $S$  which is a functional of the field values:  $S = S[\vec{\phi}]$ .

$$S: \vec{\phi} \in T \rightarrow S[\vec{\phi}] \in R$$

And the action is defined as the integral of a local Lagrangian density of the form

$$S[\vec{\phi}] = \int d^d x L(\vec{\phi})$$

where locality means that  $L$  is a function of the field, of its derivatives up to a finite order and possibly of the space time coordinates. Usually we consider only a Lagrangian which

contains derivatives up to first order. By the action principle we obtain field equations which are second order partial differential equations.

$$\delta S = 0 : \delta \vec{\phi} = 0 \text{ at space-time boundary}$$

Here, let's consider the total energy associated with a given field configuration. It can be defined directly within the Lagrangian formalism or by going to the Hamiltonian formalism via the Legendre transformation in a standard fashion. Only field configurations which yield a finite total energy will be considered as physically acceptable. In the absence of gauge degrees of freedom, this requires that the field go to some constant value at space-time infinity. Under this situation, the space-time manifold  $M = R^{d+1}$  can be considered as being compactified (via one-point compactification) to the  $(d+1)$ -sphere  $S^{d+1}$ . Therefore, the field configurations will be classified in a natural way by the homotopy group  $\Pi_{d+1}(T)$ , in the case of static field configurations by  $\Pi_d(T)$ . When the space of field configurations are decomposed into sectors classified by a homotopy group, then :

- (1) The different sectors are disjoint, i.e., the space of field configurations fall into the disjoint union of different homotopy sectors.
- (2) Infinitesimal variations of the field configurations, such as those involved in the variational principle in deriving the field equations from the action principle, cannot lead from a given homotopy sector to a different one. In other words, the field equations are independent of which homotopy sector we are in. Also, the time evolution, being a smooth process by assumption, cannot lead outside of any preassigned homotopy sector. [2]

These topologically disconnected field configurations are labelled by a geometrical quantity called the topological charge. Suppose that we add to the action a topological term, i.e., some functional of the field which depends only on the homotopy sector that the field is in. It is clear that any such term will be insensitive to smooth (and small) modifications of



the fields, and therefore the action modified in this way will lead to the same field equations. It is expected that any such term can be represented by the addition of a total divergence to the Lagrangian. Therefore, the addition of topological terms to an action does not modify the equation of motion at the purely classical level. But it can have profound consequences at the quantum level. The simplest way to see this is the quantization procedure of the Feynman path-integral in which each field configuration is weighted by the "Feynman integral factor"  $\exp[iS[\vec{\phi}]/\hbar]$ . The addition of topological terms to the action will lead to quantum interference effects between the contributions to the path-integral coming from different homotopy sectors. Such effects will be altogether absent at the classical level, where only the stationary points of the action functional in the space of field configurations will matter.

Let us begin with the following energy functional  $E$ , for static field in  $d$  space dimensions [3]:

$$E = \int d^d x \tilde{E} \equiv T_\phi[\phi, W] + T_W[W] + V[\phi]$$

$$\tilde{E} = \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} (D_\mu \phi, D^\mu \phi) + V(\phi) \quad (1)$$

Since the integrands are non-negative, each of these terms must converge separately for a finite-energy solution. Under the scale transformation;

$$\phi(x) \rightarrow \phi_\mu(x) = \phi(\mu x)$$

$$W(x) \rightarrow W_\mu(x) = \mu W(\mu x)$$

We find that

$$D_\mu \phi(x) \rightarrow \mu D_\mu \phi(\mu x)$$

$$F_{\mu\nu}(x) \rightarrow \mu^2 F_{\mu\nu}(\mu x)$$

Consequently :

$$T_\phi[\phi_\mu, W_\mu] = \mu^{2-d} T_\phi[\phi, W]$$

$$T_w[W_\mu] = \mu^{4-d} T_w[W]$$

$$V[\phi_\mu] = \mu^{-d} V[\phi]$$

For a static solution E must be stationary with respect to arbitrary field variations and therefore, in particular, the scale transformation of equations. This is certainly not the case when the terms in Eq.(1) are either all increasing or all decreasing when  $\mu$  increases. The energy may remain constant only if some of the terms in Eq.(1) are decreasing while the others are increasing, or else if these terms are independent of  $\mu$ . The behavior of the terms in Eq.(1) with the dimension of space is summarized as follows :

d	$T_\phi$	$T_w$	V
1	↑	↑	↓
2	0	↑	↓
3	↓	↑	↓
4	↓	0	↓
$d \geq 5$	↓	↓	↓

↑ indicates that the term increases as  $\mu$  increases, ↓ indicates that the term decreases as  $\mu$  increases, 0 that it is independent of  $\mu$ .

Thus we have the following cases together with the examples:

For  $d=1$  V must be present : Sine-Gordon theory,  $\phi^4$ -theory .

For  $d=2$  either all terms must be present : Higgs model-Vortex lines, or  $T_\phi$  alone :  $O(n)$  and  $CP^{n-1}$  models.

For  $d=3$  all three terms must be present : Higgs model containing monopole solutions.

For  $d=4$  the only possibility is a pure gauge theory, e.g., Instanton solution.

For  $d \geq 5$  there are no possibilities within this class of theories.

## 2.2. Symmetry and G-spaces. [4]

The G-space referred to is a space in which symmetries are relevant . It is a space  $M$  on which a symmetry group acts in a "nice" way. The group  $G$  should be a compact Lie group and the space  $M$  a smooth manifold. That the group acts in a "nice way" means that for every  $g \in G$ , the corresponding action  $f_g$  represents a diffeomorphism on  $M$ . Further, for  $g, h \in G$  and  $x \in M$  with

$$f : G \times M \rightarrow M$$

$$(g, x) \rightarrow f(g, x) = g.x = f_g(x), \quad h(g.x) = (h.g) x \text{ and } 1.x = x \text{ should be valid,}$$

where  $1$  is the unit element of the group.

One simple example for a G-space is a global flow on a manifold  $M$

$$f : \mathbb{R} \times M \rightarrow M \text{ (with } G = \mathbb{R}\text{)}.$$

The trajectory of a point  $x \in M$  is an orbit  $Rx$  of the  $\mathbb{R}$ -action. A more familiar example is the  $U(1)$  action (i.e., rotation) on the vector space  $\mathbb{R}^2$ :

$$f : U(1) \times \mathbb{R}^2 \rightarrow \mathbb{R}^2,$$

where

$$f_\phi = \begin{pmatrix} \cos \phi & -\sin \phi \\ \sin \phi & \cos \phi \end{pmatrix}.$$

We have two kinds of orbits here :

$$\text{for } x \in R^2 \text{ and } x \neq 0, U(1) \vec{x} \approx S^1$$

and

$$U(1) \vec{0} \approx \vec{0}.$$

The space of orbits  $R^2 / U(1)$  is therefore the union of the zero point and the positive line  $R^+$ .

This is shown symbolically in Fig. 2.1.

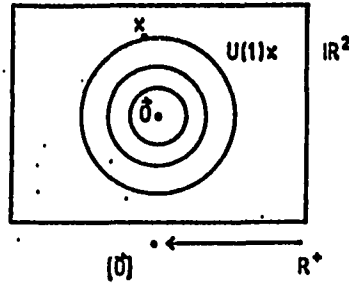


Fig. 2.1 The space of orbits  $R^2 / U(1)$

### 2.3. Spontaneous Symmetry Breaking and Vacuum Structure [5].

The ground state (vacuum) in a field theory with a symmetry  $G$ , which is spontaneously broken, is a very important example of a  $G$ -space. Spontaneous symmetry breaking is present whenever the ground state of the theory is degenerate. This is essentially described by the states of a spin zero field  $\vec{\phi}$  which minimizes the scalar potential  $V_\phi$ . This is shown in Fig. 2.2. On the left-hand-side of the figure the graph of the scalar potential is shown where  $\phi$  can be taken as the absolute value of the complex field. On the right-hand-side the " $\phi$ -points in the  $\phi$ -space" are shown, which correspond to the minimum of the potential. Since all the points are equivalent, any one of them can be chosen as the actual vacuum, and therefore we are talking of the degeneracy of the ground state.

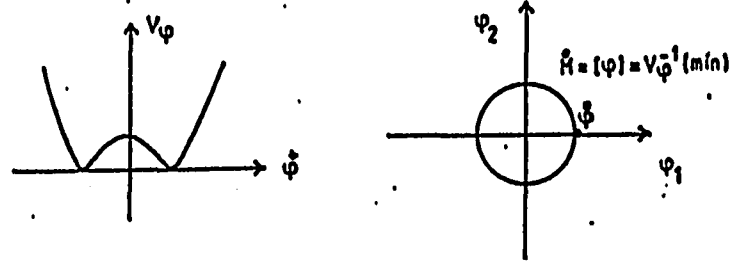


Fig. 2.2 Higgs potential

Because of the symmetry  $G$  existing in our theory, the group  $G$  acts also on the ground states  $\dot{M}$ . So  $\dot{M}$  becomes a  $G$ -manifold. There are the following properties of this construction :

- 1)  $\dot{M}$  is the only orbit of the  $G$ -action. This means that from a point  $\dot{\phi} \in \dot{M}$  the group  $G$  generates the whole space  $\dot{M}$  and we have  $\dot{M} = G\dot{\phi}$ .
- 2) There exists a maximal subgroup  $H$  ( $H \leq G$ ) so that  $H\dot{\phi} = \dot{\phi}$ .  $H$  is called as the stability group of  $\dot{\phi}$ .
- 3) This  $H$  characterizes the orbit  $M$ . This means that the stability group  $H_1$  of the element  $\phi_1 \in \dot{M}$  where  $\phi_1 \neq \dot{\phi}$  looks almost like the group  $H$ : We have from  $H_1\phi_1 = \phi_1$  and  $\phi_1 = g\dot{\phi}$   $H_1 = gHg^{-1}$ . Since  $H$  and  $H_1$  are of the same type, we can say the orbit  $\dot{M}$  is of the orbit type  $(H)$ .
- 4) There exists a model of  $\dot{M}$  :  $\dot{M} \cong G/H$ . This model produces the group  $G$  together with its subgroup  $H$ . The pair  $G$  and  $H$  contains all the secrets of  $\dot{M}$ .

The homogeneous space  $G / H$  is the space of orbits of the  $H$ -action on a space  $G$ . The elements of  $G / H$  are shown in Fig. 2.3.

The equivalence relation  $\sim$  which characterizes the elements of  $G / H$  is given by

$$g_1 \sim g_2 \Leftrightarrow g_1^{-1} g_2 \in H \Leftrightarrow g_2 = g_1 h \text{ with } h \in H.$$

Now,  $G / H$  is itself a  $G$ -space, since we can define

$$G \times G / H \rightarrow G / H$$

$$(g, g_1 H) \mapsto g(g_1 H) = g g_1 H.$$

With  $G / H$  we have constructed a second  $G$ -space. This  $G$ -space is isomorphic to  $\dot{M}$  and we have  $\dot{M} \cong G / H$ .

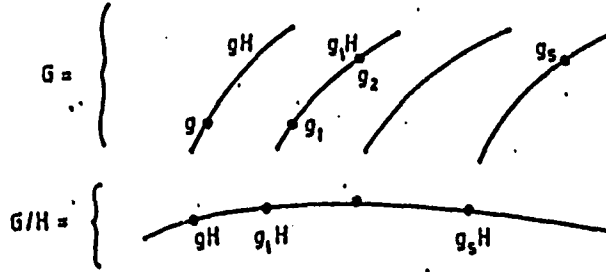


Fig. 2.3 Space  $G / H$ .

The indication of the orbit type characterizes the way the symmetry is spontaneously broken:

- a) When  $(G)$  is the orbit type of  $\dot{M}$ , then  $\phi$  is a fixed point of  $G$ . In this case there is no symmetry breaking at all since  $\dot{M} \cong G / G = \{1\}$ .
- b) When  $(1)$  is the orbit type of  $\dot{M}$ , then the action is free, so that  $\dot{M} \cong G / 1 = G$  and the symmetry is totally broken.

c) When  $(H)$  is the orbit type of  $\dot{M}$ , when  $H < G$ , then  $\dot{M} \cong G / H$  and the symmetry is broken down to the group  $H$ .

In the case of the chiral symmetries, for example, the case c) is realized. We have (for two flavors)

$$G = SU(2)_L \otimes SU(2)_R,$$

$$H = SU(2)_{L+R},$$

and the ground state is isomorphic to the sphere  $S^2$  given by

$$\frac{SU(2)_L \otimes SU(2)_R}{SU(2)_{L+R}} = G / H.$$

When the gauge group  $G$  is spontaneously broken to some residual gauge group  $H$ , the Higgs vacuum manifold satisfies the relation

$$M_0 = G / H = \text{the set of zeros of } V(\phi).$$

If we define  $\phi^\infty = \lim_{|x| \rightarrow \infty} \phi(x)$ , then we can consider  $\phi^\infty$  as a function from  $S^{D-1}$  to the manifold  $M_0$  in the  $D$ -space dimension, i.e.,

$$\phi^\infty : S^{D-1} \rightarrow G / H,$$

for  $\phi \in C$  where  $C$  is the space of static, finite energy  $E$  configurations. We have to consider the gauge orbits  $\bar{C}$  due to the complications arising from gauge invariance. The topology of  $\bar{C}$  is characterized by the topological properties of the space of  $\{\phi^\infty\} = \text{Maps}(S^{D-1}, G / H)$ . The most important topological characteristics of  $\phi^\infty$  are its homotopy groups. If there are non-trivial mappings, that is if  $\Pi_{D-1}(G / H)$  is non-trivial, then these models exhibit stable topological excitations (Vortices, Monopoles etc.).

We can summarize the properties of the topological structure associated with three possibilities.

(1) Structure by  $\Pi_{D-1}(G/H)$  : Scalar field theories in  $D = 1$ .

The Higgs model possibilities in  $D = 2, 3$ .

Pure gauge theories in  $D = 4$ .

(2) Structure by  $\Pi_D(G/H)$  : Non-linear scalar field theories for  $D \geq 2$ .

From (1) and (2), we conclude that there are two types of topological conservation laws. In the cases where gauge fields play a role, the topological structure depends on non-trivial asymptotic behavior leading to an element of a group  $\Pi_{D-1}(X)$  defined by a map from the sphere at infinity,  $S^{D-1}$ . In the scalar field cases, the topological structure depends on trivial asymptotic behavior, enabling the space to be compactified from  $R^D$  to  $S^D$  and an element of a group  $\Pi_D(X)$  to be defined by the values of the field through space. These two possibilities generally represent a stable state, in other words, they represent a minimum points of energy functional which satisfies the field equations obtained from the variational principle [6].

(3) The third possibility comes from the non-trivial  $\Pi_N(G/H)$ ,  $N > D-1$  from the first possibility. The configurations obtained from the analysis of this group correspond to an unstable classical field which represents a saddle point of the energy functional in the configuration space, not a minimum.

As it was shown by Taubes in 1983[7], the higher homotopy groups of  $\bar{C}$  are also important from both the mathematical and the physical point of view. Analyzing  $\Pi_n(\bar{C})$ , Taubes deduced the existence of a saddle point of the energy functional in the vacuum sector for an  $SU(2)$  theory broken down to  $U(1)$ . Thus the possible existence of a Taubes-type solution is signaled by the non-trivial homotopy groups  $\Pi_n(G/H) \cong \mathbb{Z}$ ,  $n > D-1$  even for the trivial  $\Pi_{D-1}(G/H)$ . These types of solutions in general do not correspond to stable configurations physically because they do not correspond to a minimum points in the field



configuration space, and are unstable under a certain perturbation. We call this solution a sphaleron solution to the corresponding model.

In the next chapter we will study the connection of the topological structure of the field configuration space to the existence of sphaleron in more detail.

## CHAPTER 3. SPHALERON SOLUTIONS

### 3.1. Topology and Non-Contractible Loop

In the preceding chapter we studied the characteristics of the finite energy static configurations in field theories in various space dimensions. The main idea is that a non-trivial topological structure of the vacuum gives rise to the existence of stable solutions as indicated by the non-trivial homotopy groups [8]. These types of solutions are topological solutions which are very stable because of the topological conservation law arising from boundary conditions imposed. Examples are kink (anti-kink) solutions in  $\phi^4$  - theory and soliton solutions in Sine- Gordon model, etc. What happens if  $\Pi_{D-1}(\bar{C})$  is trivial? Of course, there is no stable topological solution in that model. But by studying  $\Pi_D(\bar{C})$  we can predict some interesting solutions in the vacuum sector, particularly in the Standard Model. From the Morse theory[9], it is well known that there is a close connection between the topological type of critical points of a function and the topological structure of the manifold on which the function is defined. Let's consider the classical situation of the function  $\phi(x,y,z) = z$  defined on 2-dimensional torus  $T^2$  and 2-sphere  $S^2$  in  $R^3$  tangent, say, to the xy plane. The function  $\phi$  has critical points, which are the extrema in both cases. The existence of these critical points largely depends on the fact that the manifold is compact. But there are some differences in the character of the critical points in the two cases. There are 4 critical points  $u_1, u_2, u_3$  and  $u_4$  in  $T^2$  and 2 critical points  $v_1, v_2$  in  $S^2$ .  $u_4$  corresponds to a maximum point,  $u_1$  corresponds to a minimum and  $u_2, u_3$  are saddle points in  $T^2$  (Fig.3.1). In  $S^2$  there also exists a minimum and a maximum point but not the saddle type critical points. What makes this difference and how can we predict the existence of the saddle type critical point in a

manifold? The answer to these questions lies in the difference of the connectedness of the two manifolds  $T^2$  and  $S^2$ . The space  $S^2$  is simply connected but  $T^2$  is not. To see this point clearly let's construct a closed loop on  $S^2$  starting from  $v_1$  and returning to  $v_1$ . It is easy to check that there is only one type of closed loop in  $S^2$  and those loops can be shrunk to  $v_1$  continuously. In other words, all closed loops in  $S^2$  can be shrunk to a point. But in  $T^2$  we can construct two types of closed loops  $C_1$  and  $C_2$  from  $u_1$ . It is obvious that we can not deform  $C_1$  to  $C_2$  continuously, i.e.,  $C_1$  can be shrunk to  $u_1$  but  $C_2$  can not. We call  $C_2$  a Non-Contractible Loop and it plays an important role to indicate the existence of the saddle type critical point in the manifold [6].

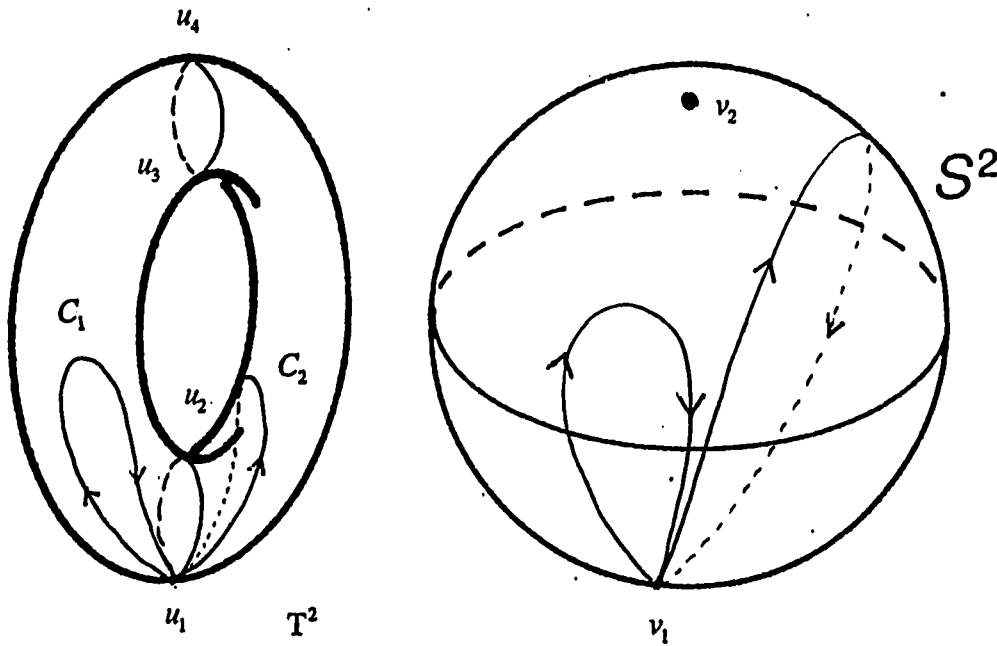


Fig. 3.1 Critical points in the manifold  $T^2$  and  $S^2$

To the manifold like  $T^2$ , we can apply the Ljusternik-Snirelman's minimax process [11] to find the saddle point in the manifold. This process was applied by Taubes in 1982 to the infinite dimensional configuration space to predict the existence of the non-minimal finite action solution in the SU(2)-Higgs model with adjoint Higgs field. And it was extended to the Standard Model by N. S. Manton in 1983 [12]. We will apply his idea to the field in general and concentrate on the spontaneously broken gauge theories. To illustrate, let us consider the following simple example of a mountain pass.

If  $\phi \in C^1(R^2, R)$ , we can view  $\phi(x, y)$  as the altitude of the point of the graph of  $\phi$  having  $(x, y)$  as projection on  $R^2$  (Fig.3.2). Assume that there exist points  $u_0 \in R^2$ ,  $u_1 \in R^2$  and a bounded open neighborhood  $\Omega$  of  $u_0$  such that  $u_1 \in R^2 \setminus \Omega$  and  $\phi(u) > \max(\phi(u_0), \phi(u_1))$  whenever  $u \in \partial\Omega$  (that is the case, for example, if  $u_0$  and  $u_1$  are two isolated local minimums of  $\phi$ ), where  $\partial\Omega$  represents the boundary of the region  $\Omega$ . Looking at a topographic plot of  $\phi$ , we can consider the point  $[u_0, \phi(u_0)]$  as located in a valley surrounded by a ring of mountains pictured by the set  $\{[u, \phi(u)] : u \in \partial\Omega\}$ . we approach the point  $[u_1, \phi(u_1)]$  in a way which minimizes the highest altitude on the path; we must cross the mountain ring through the lowest mountain pass. The projection on  $R^2$  of the top of this mountain pass will provide a critical point of  $\phi$  with critical value

$$c = \inf_{g \in \Gamma} \max_{s \in [0,1]} \phi(g(s))$$

where  $\Gamma$  denotes the set of homotopic paths joining  $u_0$  to  $u_1$  (i.e., the set of continuous mappings  $g: [0,1] \rightarrow R^2$  with  $g(0) = u_0$ ,  $g(1) = u_1$ ). From this argument it is obvious that the actual point we have to take to pass over the mountain is a saddle point with respect to the height. In field theory, this saddle point corresponds to an unstable physical static state which sets the energy scale that is needed to go from one minimum state to the other classically when there are many relative minimum points in the manifold under consideration.

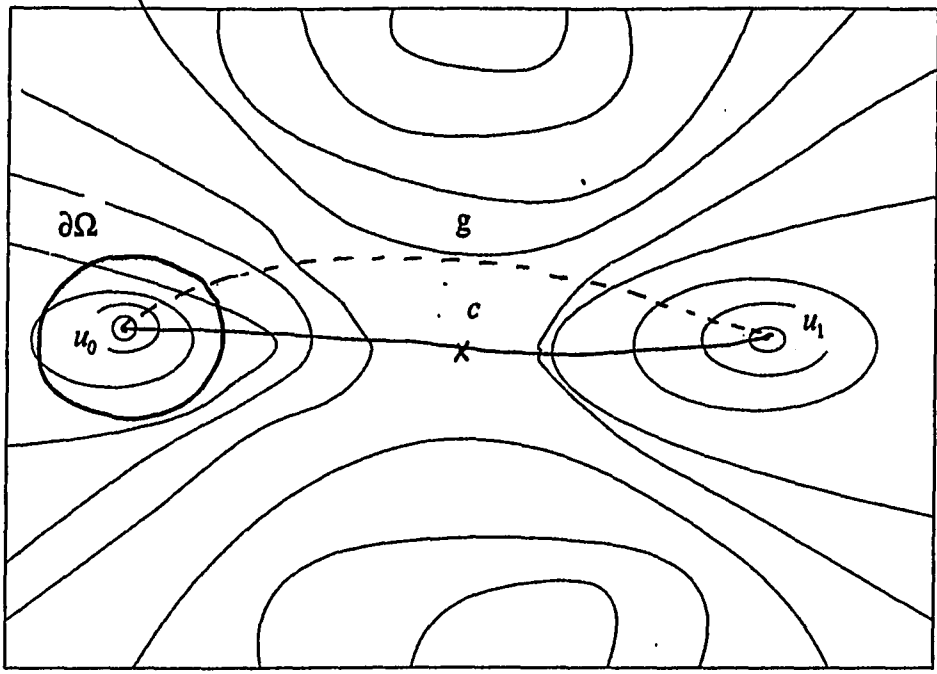


Fig. 3.2 Topographic view of the saddle point in the mountain pass

Therefore the state we are looking for actually corresponds to the configuration which satisfies the relation on the NCL :  $\min(\max(E|_{NCL}))$ .

Let's consider a 1+1-dimension problem as an example [13]. The Lagrangian is given by

$$L = \frac{1}{2} \partial_0 \phi \partial^0 \phi^* - \left[ \frac{1}{2} \partial_1 \phi \partial^1 \phi^* + \frac{1}{2} \lambda (\phi \phi^* - 1)^2 \right]$$

and the static energy by

$$E = \int dx \left[ \frac{1}{2} \partial_1 \phi \partial^1 \phi^* + \frac{1}{2} \lambda (\phi \phi^* - 1)^2 \right].$$

Since  $\Pi_0(S^1) \cong 0$ , there are no topologically stable, static, finite energy solutions in this model. Here  $\phi^*: [-\infty, \infty] \rightarrow S^1$ . Now let's consider a family of maps,  $\phi^*(\mu)$  which

depends on  $\mu$  continuously in the range  $0 \leq \mu \leq 2\pi$ , beginning and ending at the same vacuum state, denoted as  $\phi_{vac}=1$ . With such a family, we associate a loop in the configuration space  $\phi(\mu, x)$ . The asymptotic configurations on the loop are given by  $\phi^\infty(\mu) : \{[-\infty], [\infty]\} \rightarrow S^1$ ,  $\phi^\infty(0) = \phi^\infty(2\pi) = 1$ . The crucial point is that the family of maps  $\phi^\infty(\mu)$  is topologically equivalent to a single map  $\Psi: S^1 \rightarrow S^1$ . This means that  $\phi^\infty(\mu)$  is the generator of  $\Pi_1(S^1) \cong \mathbb{Z}$ . The fact that  $\phi^\infty(\mu)$  is a non-trivial  $S^1 \rightarrow S^1$  map guarantees that the loop in  $\mathcal{C}$  is a NCL. Let's take the simplest map whose degree is 1, i.e.,  $\phi^\infty(\mu) = e^{i\mu}$ . Take  $\phi(\mu, x) = f(x) \exp(i\mu) + 1 - f(x)$ ,  $0 \leq \mu \leq 2\pi$  with the boundary conditions  $f(x) \xrightarrow{x \rightarrow \infty} 1$  and  $f(x) \xrightarrow{x \rightarrow -\infty} 0$ . Then the energy functional is given by

$$E(\mu) =$$

$$\int dx \left[ \frac{1}{2} (f' \exp(-i\mu) - f') (f' \exp(i\mu) - f') + \frac{1}{2} \lambda [(f \exp(-i\mu) + 1 - f)(f \exp(i\mu) + 1 - f) - 1]^2 \right]$$

$$= \frac{1}{2} \int dx \{ 2f'^2 (1 - \cos \mu) + \lambda [f^2 + (1 - f)^2 + 2f(1 - f) \cos \mu - 1]^2 \}.$$

The derivative of the energy functional vanishes at  $\mu = \pi$ .

$$\frac{\partial E}{\partial \mu} = \int dx \{ f'^2 \sin \mu - 4\lambda f^2 (1 - f)^2 (\cos \mu - 1) \sin \mu \} = 0.$$

It is easy to see that  $\frac{\partial^2 E}{\partial^2 \mu} \geq 0$  then

$$E_{\max} = 2 \int dx \{ f'^2 + 4\lambda f^2 (1 - f)^2 \}.$$

By minimizing this energy functional  $\delta E_{\max} = 0$ , we get the equation of motion for the unstable configuration :

$$f'' - 4\lambda f(1 - f)(1 - 2f) = 0.$$

The general solution is

$$f = c_1 \exp(-2\sqrt{\lambda}r) + c_2 \exp(2\sqrt{\lambda}r).$$

Choose

$$f = \frac{\exp(-2\sqrt{\lambda}r)}{\exp(-2\sqrt{\lambda}r) + \exp(2\sqrt{\lambda}r)} \text{ to get } \phi_0 = \tanh(2\sqrt{\lambda}r).$$

The configuration space of an electroweak theory is an infinite dimensional, non-compact manifold. This enormous space is difficult to visualize, but the energy surface over one particular slice is sketched in Fig.3.3.

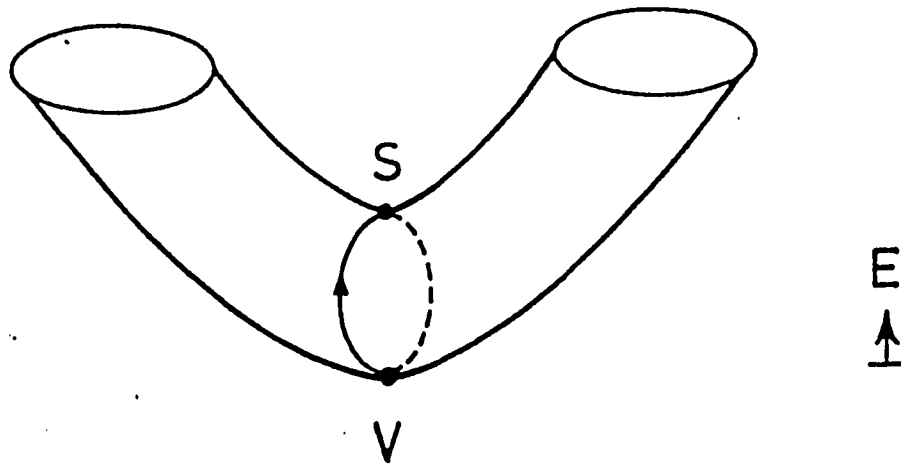


Fig.3.3 Energy surface in E-W theory

By adopting Taubes' rigorous proof of the existence of the saddle point type solution, We explicitly construct the NCL in the configuration space of the Standard Model. The gauge potential is an anti-Hermitian 2x2 matrix and the Higgs field is a two-component complex vector

$$\begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix}.$$

We denote by  $\phi_{Re}$  the four-component real vector  $\phi_{Re} = (\text{Re } \phi_1, \text{Im } \phi_1, \text{Re } \phi_2, \text{Im } \phi_2)$ . In the polar coordinate  $\{r, \theta, \varphi\}$ , the associated covariant components of the gauge potential  $\{A_r, A_\theta, A_\varphi\}$  are related to the Cartesian components by

$$A_r dr + A_\theta d\theta + A_\varphi d\varphi = A_i dx^i.$$

We are interested in fields which are smooth in their Cartesian form. Let us impose the polar gauge condition  $A_r = 0$ . Any field configuration with  $A_r \neq 0$  can be put in this gauge via the gauge transformation  $U(r, \theta, \varphi) = P \exp[\int_0^1 A_r(\sigma r, \theta, \varphi) r d\sigma]$ . There is no further local gauge freedom; the gauge functions are uniquely defined with the choice  $A_r = 0$ . In particular, a gauge transformation  $U(\theta, \varphi)$  which is independent of the radial coordinate would preserve the polar gauge condition, but this is ill-defined at the origin and it leads to a singular gauge potential there, unless  $U$  is a constant, independent of  $\theta$  and  $\varphi$ . Asymptotically, the magnitude of the Higgs field of a finite-energy configuration must tend to 1. We suppose that in the polar gauge there exists a limiting field  $\phi^\infty(\theta, \varphi) = \lim_{r \rightarrow \infty} \phi(r, \theta, \varphi)$  which is a smooth function of  $\theta$  and  $\varphi$ , and which satisfies

$$|\phi^\infty| = 1, \quad (a)$$

where we factored out the constant factor  $\frac{v}{\sqrt{2}}$ .



Let us use the global gauge freedom to fix  $\phi^{\sim}(\theta=0) = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ . The vacuum configuration is completely fixed. It is

$$\phi_{vac}(x) = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, A_{vac} = 0.$$

Eq. (a) implies that we can regard  $\phi_{Re}^{\sim}$  as a map  $\phi_{Re}^{\sim}: S^2 \rightarrow S^3$ , where  $S^2$  is the 2-sphere at spacial infinity and  $S^3$  is the vacuum manifold of the Higgs field. The homotopy group  $\Pi_2(S^3)$  is trivial, so the map  $\phi_{Re}^{\sim}$  is contractible. It follows that any finite-energy field configuration can be continuously transformed to the vacuum, and for this reason there are no magnetic monopoles in the Weinberg-Salam theory. We are interested in loops in the configuration space, beginning and ending at the vacuum. Let  $\mu \in [0, \pi]$  be the parameter along one such loop. The asymptotic Higgs fields of the configuration on the loop define a family of maps  $\phi_{Re}^{\sim}: S^2 \rightarrow S^3$ , varying continuously with  $\mu$ .  $\phi_{Re}^{\sim}(0)$  and  $\phi_{Re}^{\sim}(\pi)$  are identical constant maps, mapping all of  $S^2$  to the point  $(0,0,1,0)$  on  $S^3$ , because they correspond to vacuum configurations. For all  $\mu$ ,  $\phi_{Re}^{\sim}$  maps the point  $(\theta=0)$  on  $S^2$  to the point  $(0,0,1,0)$  on  $S^3$ , because of our gauge choice. The point is that such a family of maps is topologically equivalent to a single map  $\Psi: S^3 \rightarrow S^3$ . We associate with each triplet  $(\mu, \theta, \varphi)$  a point  $p(\mu, \theta, \varphi)$  on  $S^3$ , which is written as a four-component unit vector

$$p(\mu, \theta, \varphi) = (\sin\mu \sin\theta \cos\varphi, \sin\mu \sin\theta \sin\varphi, \sin^2\mu \cos\theta + \cos^2\mu, \sin\mu \cos\mu (\cos\theta - 1)).$$

This assignment has the following desired properties :

- (1)  $p(\mu, \theta, \varphi)$  is continuous in its argument.
- (2)  $p$  is unchanged under  $\varphi \rightarrow \varphi + 2\pi$ , and  $p$  is independent of  $\varphi$  when  $\theta=0, \pi$ .
- (3) For all  $\mu$ ,  $p(\mu, \theta=0, \varphi) = (0,0,1,0)$

(4) For  $\mu = 0$  and  $\mu = \pi$ ,  $p(\mu, \theta, \varphi) = (0, 0, 1, 0)$  for all  $\theta, \varphi$ .

(5) For each point  $p$  on  $S^3_{dom}$  there occurs for at least one triplet  $(\mu = \mu(p), \theta = \theta(p), \varphi = \varphi(p))$  and if  $p$  is not the point  $(0, 0, 1, 0)$ , then  $\mu(p)$  is unique ( $0 < \mu(p) < \pi$ ) and  $(\theta(p), \varphi(p))$  represents a unique point of  $S^2$ . The map  $\Psi$  may now be defined by

$$\Psi(p) = \phi_{Re}^{\infty}(\mu(p), \theta(p), \varphi(p)).$$

We have now associated with a loop in the configuration space, beginning and ending at the vacuum, a map  $\Psi: S^3 \rightarrow S^3$ .  $\Psi$  is important because the degree of the function is a topological property. A simple map of nonzero degree is the identity map, which has degree one. With this choice of  $\Psi$ , the asymptotic Higgs fields are

$$\phi^{\infty}(\mu, \theta, \varphi) = \begin{bmatrix} \sin \mu \sin \theta e^{i\varphi} \\ e^{-i\mu} (\cos \mu + i \sin \mu \cos \theta) \end{bmatrix}.$$

A loop of finite-energy field configurations whose asymptotic Higgs fields are given as above will be noncontractible (Fig. 3.4). A suitable ansatz for the asymptotic gauge potential is

$$A_{\{\theta\}}^{\infty} = -\partial_{\{\theta\}} U^{\infty} (U^{\infty})^{-1}.$$

Here  $U^{\infty}$  is a  $SU(2)$  matrix

$$U^{\infty} = \begin{pmatrix} \phi_2^{\infty*} & \phi_1^{\infty} \\ -\phi_1^{\infty*} & \phi_2^{\infty} \end{pmatrix}.$$

Consider the following field configurations:

$$\phi(\mu, r, \theta, \varphi) = [1 - h(r)] \begin{bmatrix} 0 \\ e^{-i\mu \cos \mu} \end{bmatrix} + h(r) \phi^{\infty}(\mu, \theta, \varphi)$$

$$A_{\left\{\begin{smallmatrix}\theta\\\varphi\end{smallmatrix}\right\}}(\mu, r, \theta, \varphi) = f(r) A_{\left\{\begin{smallmatrix}\theta\\\varphi\end{smallmatrix}\right\}}^{\sim}(\mu, \theta, \varphi)$$

$$A_r(\mu, r, \theta, \varphi) = 0.$$

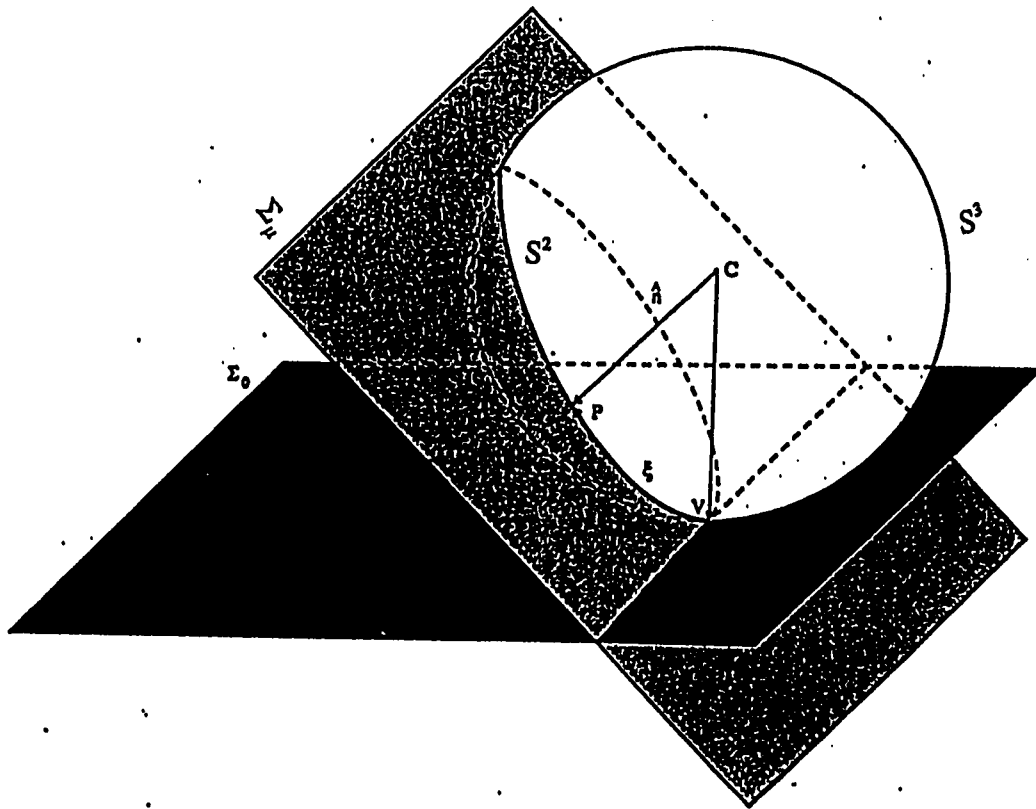
These fields are defined over all of space. They are smooth and have finite energy for suitable radial functions  $f$  and  $h$ . Here  $f$  and  $h$  must satisfy the boundary conditions

$$\begin{aligned} \lim_{r \rightarrow 0} h(r) &= 0, \quad \lim_{r \rightarrow \infty} h(r) = 1 \\ \lim_{r \rightarrow 0} \frac{1}{r} f(r) &= 0, \quad \lim_{r \rightarrow \infty} f(r) = 1 \end{aligned}$$

to ensure smoothness at the origin and to ensure that the fields have the desired asymptotic behavior. For  $\mu = 0$  and  $\mu = \pi$ , the fields are those of the vacuum. We conclude that the fields represent a noncontractible loop in the configuration space of the classical Weinberg-Salam theory, beginning and ending at the vacuum. Let us note that by relaxing the polar gauge condition a noncontractible loop connecting the vacuum to itself can be cast in a form where it becomes a path which connects "topologically distinct" vacuums. If one imposes the gauge condition that at spatial infinity all fields must approach the unitary vacuum, and also that the configuration corresponding to  $\mu = 0$  is still the unitary vacuum, then the configuration corresponding to  $\mu = \pi$  will have the form

$$\phi(x) = U(x) \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \quad A_i(x) = -(\partial_i U U^{-1})(x),$$

where  $U \rightarrow 1$  at spatial infinity, and  $U$  cannot be continuously transformed to  $U \equiv 1$  while preserving this boundary condition.



$$\Sigma_\mu = p_3 \cos \mu - p_4 \sin \mu = \cos \mu$$

Fig. 3.4. Construction of the Non-Contractible Loop

### 3.2. SU(2) Sphaleron [15 - 36]

The bosonic fields of the standard Glashow-Weinberg-Salam theory are the SU(2) $\times$ U(1) gauge fields  $A_\mu^a, B_\mu$   $a = 1, 2, 3$  and the complex Higgs doublet  $\phi^+ = (\phi^-, \phi^{0*})$ . The Lagrangian of the bosonic sector is

$$L = L_A + L_B + L_\phi,$$

where

$$L_B = -\frac{1}{4} B_{\mu\nu} B^{\mu\nu}, \quad L_A = -\frac{1}{4} \text{Tr} F_{\mu\nu} F^{\mu\nu}, \quad L_\phi = (D_\mu \phi)(D^\mu \phi) - \lambda(\phi\phi - \frac{v^2}{2})^2$$

$$D_\mu \phi(x) = (\partial_\mu - \frac{1}{2} ig \underline{\tau} \cdot A_\mu - \frac{1}{2} ig' B_\mu) \phi(x)$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - \frac{1}{2} ig [A_\mu, A_\nu]$$

and

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu.$$

Let's define the matrix

$$M(x) \equiv (\tilde{\phi}, \phi) = \begin{pmatrix} \phi^{0*}(x) & \phi^+(x) \\ -\phi^-(x) & \phi^0(x) \end{pmatrix},$$

where  $\tilde{\phi}(x) = i\tau_2 \phi^*(x)$ . Under a local  $SU(2)_L \times U(1)$  gauge transformation

$$\phi(x) \rightarrow \phi'(x) = e^{i[\epsilon_0(x) + \underline{\epsilon}(x) \cdot \underline{\mathcal{T}}(x)]} \phi(x)$$

$$\tilde{\phi}(x) \rightarrow \tilde{\phi}'(x) = e^{i[-\epsilon_0(x) + \underline{\epsilon}(x) \cdot \underline{\mathcal{T}}(x)]} \tilde{\phi}(x), \quad (b)$$

from which we may deduce the transformation law of  $M(x)$  under the local  $SU(2)_L \times U(1)$  group :

$$\begin{aligned} M(x) &\rightarrow M'(x) = (\tilde{\phi}'(x), \phi'(x)) \\ &= e^{i\tilde{\epsilon}(x)\cdot\mathbf{T}(x)} (\tilde{\phi}(x), \phi(x)) e^{-i\epsilon_0(x)\cdot\tau_3} \\ &= e^{i\tilde{\epsilon}(x)\cdot\mathbf{T}(x)} M(x) e^{-i\epsilon_0(x)\cdot\tau_3} \end{aligned}$$

Of course,  $D_\mu \phi(x)$  transforms under a local  $SU(2)_L \times U(1)$  gauge transformation according to the law given in Eq. (b), which leads us to the conclusion that the covariant derivative of  $M(x)$  is

$$\begin{aligned} D_\mu M(x) &= (i\tau_2 [D_\mu \phi(x)]^* D_\mu \phi(x)) \\ &= (\partial_\mu - \frac{1}{2}ig\mathbf{T} \cdot A_\mu(x))(\tilde{\phi}(x), \phi(x)) + \frac{1}{2}ig' B_\mu(\tilde{\phi}(x), \phi(x))\tau_3. \\ &= \partial_\mu M(x) - \frac{1}{2}ig\mathbf{T} \cdot A_\mu(x)M(x) + \frac{1}{2}ig' B_\mu M(x)\tau_3 \end{aligned}$$

We can write down the minimal Weinberg-Salam model lagrangian in the absence of fermions :

$$L_{LW} = -\frac{1}{4}B_{\mu\nu}B^{\mu\nu} - \frac{1}{4}\text{Tr}(F_{\mu\nu}F^{\mu\nu}) + \frac{1}{2}\text{Tr}((D_\mu M^+(x))D^\mu M(x)) - \lambda[\frac{1}{2}\text{Tr}(M^+M) - \frac{v^2}{2}]^2.$$

let's focus on the scalar sector of the theory

$$L_{scalar} = \frac{1}{2}\text{Tr}(\partial_\mu M^+ \partial^\mu M) - \lambda[\frac{1}{2}\text{Tr}(M^+M) - \frac{v^2}{2}]^2.$$

In addition to being invariant under the global  $SU(2)_L \times U(1)$  group,  $L_{scalar}$  is invariant under the global  $SU(2)_L \times SU(2)_R$  gauge transformation

$$M \rightarrow M' = e^{i\tilde{\epsilon}_L \cdot \mathbf{T}} M e^{i\epsilon_R \cdot \mathbf{T}}.$$

Once the global  $SU(2)_L \times U(1)$  symmetry is gauged the scalar sector loses this additional global  $SU(2)_L \times SU(2)_R$  chiral symmetry, due to the presence of the  $\tau_3$  matrix in the  $U(1)$  portion of the covariant derivative. In the limit  $g' = 0$  (i.e.,  $\theta_w = 0$ ), the gauge group reduces to  $SU(2)_L$  and the chiral symmetry is restored. It is the accidental  $SU(2)_L \times SU(2)_R$  symmetry of the potential which is responsible for the phenomenologically successful, natural relation  $\rho = \left( \frac{M_w}{M_Z \cos \theta} \right)^2 = 1$  of the minimal Weinberg-Salam model.

We set  $g' = 0$  in the energy functional, i.e.,  $\theta_w = 0$ , then the  $U(1)$  gauge potential  $a_i$  decouples and may be set to zero. In this special case we expect a spherically symmetric solution. This solution is meaningful since  $\sin^2 \theta_w$  is small and the dependence on the Weinberg angle is not strong. So the omission of the  $U(1)$  is not too far from reality. Here we mean by spherically symmetric when the spatial rotation is exactly cancelled by the internal gauge transformation because we are considering the direct product space of 3-space and the weak isospin internal space. The general continuous group for this static configuration is  $SO(3) \otimes SO(3)$ , the product of spatial and isotopic rotations. We need a diagonal subgroup consisting of simultaneous and equal rotations in the real and isotopic spaces. Using the polar decomposition theorem, the field  $M(x)$  can be put into the form

$$M(x) = h(x) U(x), \quad h(x) = |M(x)| = \sqrt{\phi^\dagger \phi}, \quad U(x) \in SU(2).$$

The general ansatz of invariant with respect to the diagonal group, which has the generators  $-i\underline{J} + \underline{T}$ , satisfies :

$$\begin{aligned} -i(\underline{x} \times \nabla)_i U + [\tau_i, U] &= 0 \\ -i(\underline{x} \times \nabla)_i A_j - i\epsilon_{ijk} A_k + [\tau_i, A_j] &= 0 \end{aligned}$$

We obtain the general solution which takes the form

$$U = (\cos \theta + i \underline{\tau} \cdot \underline{x} \sin \theta)$$

$$A_i = \frac{\alpha(r)}{r} (\hat{x} \times \underline{\tau})_i + \frac{\beta(r)}{r} \tau_i + \frac{\gamma(r)}{r} (\hat{x} \cdot \underline{\tau}) \hat{x}_i$$

There are also discrete symmetries :

$$P: \phi^a(\underline{x}) \rightarrow \phi^a(-\underline{x}), A_i^a(\underline{x}) \rightarrow A_i^a(-\underline{x})$$

$$C: \phi^a(\underline{x}) \rightarrow \phi^{a*}(\underline{x}), A_i^a(\underline{x}) \rightarrow -A_i^a(\underline{x})$$

Invariance with respect to CP forces  $\beta(r) = \gamma(r) = 0$ . The solution then has an invariance group  $G_0 \cong SO(3) \times Z_2$ , the  $SO(3)$  being the diagonal group in the product of spatial and isotopic rotations and  $Z_2$  being the group generated by CP.

The field equations are

$$(D_j F_{ij})^a = -\frac{1}{2} i g [\phi^\dagger \sigma^a D_i \phi - (D_i \phi)^\dagger \sigma^a \phi]$$

$$D_i D_i \phi = 2\lambda (\phi^\dagger \phi - \frac{1}{2} v^2) \phi,$$

where

$$(D_j F_{ij})^a = \partial_j F_{ij}^a + g \epsilon^{abc} A_j^b F_{ij}^c$$

$$D_j \phi = \partial_j \phi - \frac{1}{2} i g \sigma^a A_j^a \phi$$

The general spherically symmetric solution which yields spherically symmetric distribution both of the energy and topological charge is written as

$$A_j^a(r) = \epsilon_{jam} \frac{f(gvr)}{r} x_m$$

$$\phi(r) = \frac{v}{\sqrt{2}} h(gvr) \hat{x} \cdot \hat{\tau} \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

For these fields, the energy functional becomes



$$E = \frac{4\pi\nu}{g} \int_0^{\tilde{\xi}} d\xi \left[ 4 \left( \frac{df}{d\xi} \right)^2 + \frac{8}{\xi^2} [f(1-f)]^2 + \frac{1}{2} \xi^2 \left( \frac{dh}{d\xi} \right)^2 + [h(1-f)]^2 + \frac{1}{4} \left( \frac{\lambda}{g^2} \right) \xi^2 (h^2 - 1)^2 \right].$$

Then the field equations reduce to

$$\begin{aligned} \xi \frac{d^2 f}{d\xi^2} &= 2f(1-f)(1-2f) - \frac{\xi^2}{4} h^2 (1-f) \\ \frac{d}{d\xi} \left[ \xi^2 \frac{dh}{d\xi} \right] &= 2h(1-f)^2 + \frac{\lambda}{g^2} \xi^2 (h^2 - 1)h \end{aligned}$$

From the boundary conditions on the functions  $f$  and  $h$ , i.e.,  $f, h \rightarrow 0$  as  $\xi \rightarrow 0$  and  $f, h \rightarrow 1$  as  $\xi \rightarrow \infty$ , we obtain the asymptotic behavior near  $\xi = 0$  and infinity :

$$f = \alpha \xi^2, \quad h = \beta \xi \quad \text{as } \xi \rightarrow 0$$

$$f = 1 - \gamma \exp\left(-\frac{1}{2}\xi\right), \quad h = 1 - \frac{\delta}{\xi} \exp\left(-\sqrt{\frac{2\lambda}{g^2}} \xi^2\right) \quad \text{as } \xi \rightarrow \infty.$$

Here  $\alpha, \beta, \gamma, \delta$  are constants to be determined numerically.

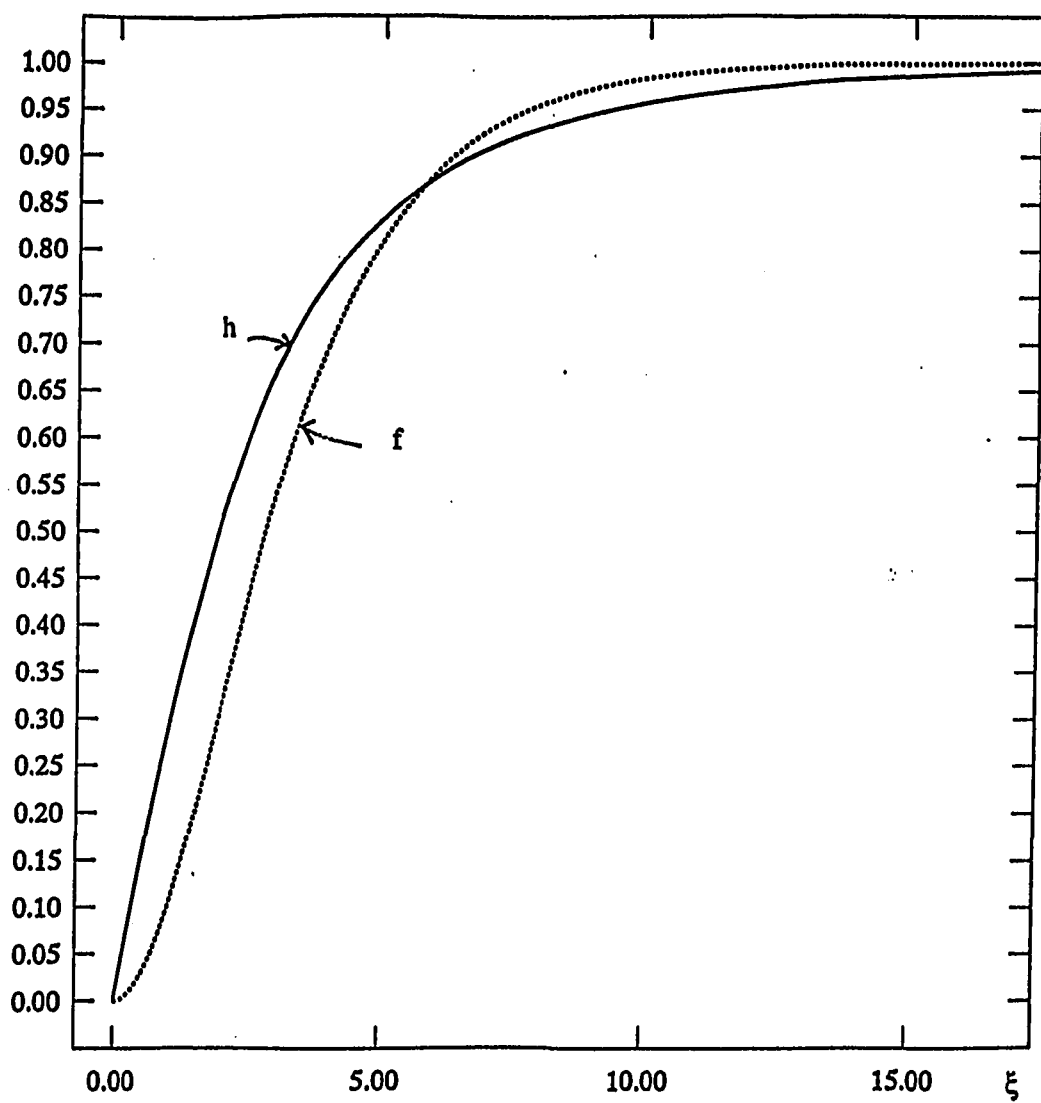


Fig. 3.5 SU(2) - Sphaleron

### 3.3. SU(2) X U(1) Sphaleron [37 - 39]

In the previous chapter we showed NCL in the SU(2)-Higgs theory. We will extend this NCL to the SU(2)XU(1) case to include the U(1) hypercharge gauge field as well.

The neutral vector boson Z and photon A is given as

$$\begin{aligned} Z &= W^3 \cos \theta_w - B \sin \theta_w \\ A &= W^3 \sin \theta_w + B \cos \theta_w \end{aligned}$$

with masses  $M_w = gv/2$ ,  $M_Z = M_w / \cos \theta_w$  and  $M_A = 0$ , where the weak mixing angle  $\theta_w$  is defined by  $\tan \theta_w = g' / g$ . As  $\theta_w \rightarrow \frac{\pi}{2}$ , the mass of the Z field becomes infinite and the Z field should be allowed to vanish rapidly in this ansatz. This condition relates the U(1) field B to the  $A^3$  field.

From the SU(2) result, we know that  $U^\infty$  provides a noncontractible mapping of  $S^1 \times S^2 \cong S^3$  into  $SU(2) \cong S^3$

$$\begin{aligned} U^\infty(v, \theta, \varphi) &= (\cos^2 v + \sin^2 v \cos \theta) I \\ &+ \sin v \cos v (1 - \cos \theta) i \sigma_3 \\ &+ \sin v \sin \theta (\sin \varphi i \sigma_1 + \cos \varphi i \sigma_2) \end{aligned}$$

where  $v \in [0, \pi]$  is a parameter of NCL and  $\theta, \varphi$  are the coordinates of the sphere at infinity.

We also define the following 1-form  $F_a$ :

$$U^{\infty-1} dU^\infty \equiv \sum_{a=1}^3 F_a \frac{\sigma^a}{2i}.$$

We obtain

$$F_1 = -2\{\sin^2 v \cos v \cos \phi + \sin^3 v \sin \phi + (\cos^2 v \sin v \sin \phi - \sin v \cos v \cos \phi) \cos \theta\} d\theta$$

$$-2\{(\cos^2 v + \sin^2 v \cos \theta) \sin v \sin \theta \cos \phi + \sin^2 v \cos v \sin \theta (1 - \cos \theta) \sin \phi\} d\phi$$

$$F_2 = -2\{\sin^2 v (\sin v \cos \phi - \cos v \sin \phi) + \sin v \cos v \cos \theta (\cos v \cos \phi + \sin v \sin \phi)\} d\theta$$

$$-2\{\sin^2 v \cos v \sin \theta (1 - \cos \theta) \cos \phi - (\cos^2 v + \sin^2 v \cos \theta) \sin v \sin \theta \sin \phi\} d\phi$$

$$F_3 = 2(\sin^2 v \sin^2 \theta d\phi - \sin v \cos v \sin \theta d\theta).$$

We are now constructing our NCL, which is parameterized by  $v \in [-\frac{\pi}{2}, \frac{3\pi}{2}]$ . The NCL starts

and ends at the vacuum and consists of three phases :

(I)  $v \in [-\frac{\pi}{2}, 0]$  build up the Higgs configuration,

(II)  $v \in [0, \pi]$  build up and destroys the gauge fields,

(III)  $v \in [\pi, \frac{3\pi}{2}]$  destroys the Higgs configurations.

Phase (I),(III) :

$$gW = g'B = 0$$

$$\phi = \frac{v}{\sqrt{2}} \begin{pmatrix} 0 \\ \sin^2 v + h \cos^2 v \end{pmatrix}, v \in [-\frac{\pi}{2}, 0] \cup [\pi, \frac{3\pi}{2}].$$

Phase (II) :

$$gW = (1-f)[F_1 \frac{\sigma_1}{2i} + F_2 \frac{\sigma_2}{2i}] + (1-f_3)[F_3 \frac{\sigma_3}{2i}]$$

$$g'B = (1-f_0)F_3$$

$$\phi = \frac{\nu}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \nu \in [0, \pi]$$

where  $f, f_3, f_0$  and  $h$  are radial functions with the following boundary conditions:

$$r = 0 : f = f_3 = h = 0, f_0 = 1$$

$$r = \infty : f = f_3 = h = f_0 = 1.$$

The energy over NCL is for phases (I) and (III)

$$E(\nu) = \frac{\nu}{g} \int \xi^2 \sin \theta d\xi d\theta d\varphi$$

$$\times [\cos^4 \nu \frac{1}{2} h'^2 + \frac{1}{4} \frac{\lambda}{g} ((1 - (1-h) \cos^2 \nu)^2 - 1)^2]$$

and for phase (II)

$$E(\nu) = \frac{\nu}{g} \int \xi^2 \sin \theta d\xi d\theta d\varphi$$

$$\times [\sin^2 \nu \frac{1}{\xi^2} \{4f'^2 + 2\sin^2 \theta (f_3'^2 - f'^2)\}$$

$$+ \sin^4 \nu \frac{8}{\xi^4} \{\sin^2 \theta f_3'^2 (1-f)^2 + \cos^2 \theta (f(1-f) + f - f_3)^2\}$$

$$+ \left(\frac{g}{g'}\right)^2 \{\sin^2 \nu \sin^2 \theta \frac{2}{\xi^2} f_0'^2 + \sin^4 \nu \cos^2 \theta \frac{8}{\xi^4} (1-f_0)^2\}$$

$$+ \frac{1}{2} h'^2 + \sin^2 \nu \frac{h^2}{2\xi^2} \{\sin^2 \theta (f_0 - f_3)^2 + (2 - \sin^2 \theta)(1-f)^2\}$$

$$+ \frac{1}{4} \frac{\lambda}{g^2} (h^2 - 1)^2],$$

where  $\xi = gvr$  is the dimensionless radial coordinate and a prime denotes differentiation with respect to  $\xi$ . The maximal energy is attained at  $v = \pi/2$ . The solution is given in Fig. 3.6.

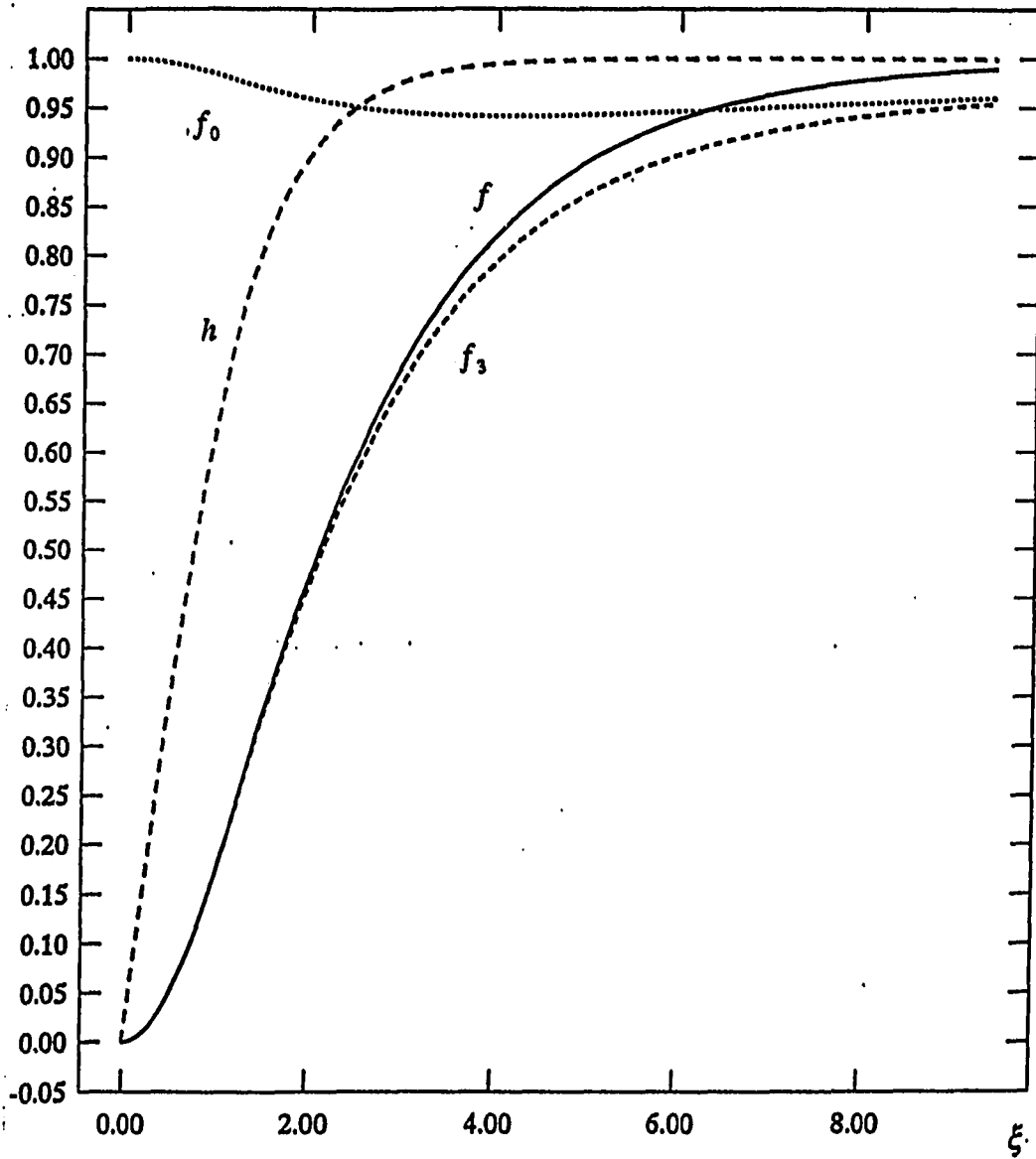


Fig. 3.6 SU(2)XU(1) - Sphaleron

## CHAPTER 4. THE SPHALERON IN EFFECTIVE THEORIES

### 4.1. Dimension 6 Operators

Beyond the standard model interactions with a scale  $\Lambda$  can manifest themselves at energies below  $\Lambda$  through deviations from the standard model, and are described by an effective Lagrangian containing  $SU(3) \times SU(2) \times U(1)$  invariant but non-renormalizable operators. In this spirit the Standard Model may be considered as an effective theory in which heavy fermion fields have been integrated out. The total Lagrangian is given by

$$L = L_0 + \frac{1}{\Lambda} L_1 + \frac{1}{\Lambda^2} L_2 + \dots$$

where  $L_0$  is the standard Lagrangian of dimension 4 ( $SU(3) \times SU(2) \times U(1)$  gauge fields, the usual fermion fields and one Higgs doublet).  $\text{Dim} [L_0] = 4$ ,  $\text{Dim} [L_1] = 5$ ,  $\text{Dim} [L_2] = 6$ , etc., where all  $L_i$  are  $SU(3) \times SU(2) \times U(1)$  invariant. We consider only  $SU(2)$  symmetric operator up to dimension 6. Let's analyze the operators corresponding to vacuum sector. There are no dimension 5 operators in the vacuum sector and we have 6 operators of dimension 6 which can be constructed from the scalar and vector field [40].

Vectors only:

$$O_W = \epsilon_{IJK} W_\mu^{I\nu} W_\nu^{J\lambda} W_\lambda^{K\mu}.$$

Scalar only:

$$O_\phi = \frac{1}{3} (\phi^\dagger \phi)^3$$

$$O_{\partial\phi} = \frac{1}{2} \partial_\mu (\phi^\dagger \phi) \partial^\mu (\phi^\dagger \phi).$$

Scalars and Vectors:

$$O_{\phi}^{(1)} = (\phi^+ \phi) ((D_{\mu} \phi)^+ D^{\mu} \phi)$$

$$O_{\phi}^{(3)} = (\phi^+ D_{\mu} \phi) ((D^{\mu} \phi)^+ \phi)$$

$$O_{\phi W} = \frac{1}{2} (\phi^+ \phi) W_{\mu\nu}^I W^{\mu\nu I}.$$

We consider the Weinberg-Salam theory in the limit of vanishing mixing angle  $\Theta_w$ . Then the U(1) field decouples. Using the spherical symmetric ansatz, we can write the static field in the  $W_0 = 0$  gauge,

$$W_i^a \sigma^a dx^i = -\frac{2i}{g_w} f(\xi) dU^{\infty} (U^{\infty})^{-1}$$

$$\phi = \frac{v}{\sqrt{2}} h(\xi) U^{\infty} \begin{pmatrix} 0 \\ 1 \end{pmatrix},$$

where  $U^{\infty} = \frac{1}{r} \begin{pmatrix} z & x+iy \\ -x+iy & z \end{pmatrix}$  and  $\xi = g_w v r$ .

Then the operators are given in terms of the function f and h as follows:

$$O_1 = \frac{1}{\Lambda^2} O_{\phi} = \frac{1}{3} \frac{1}{\Lambda^2} \frac{v^6}{2^3} (h(\xi)^2 - 1)^3$$

$$O_2 = \frac{1}{\Lambda^2} O_{\partial\phi} = \frac{1}{2} \frac{1}{\Lambda^2} g_w^2 v^6 h(\xi)^2 \left( \frac{dh}{d\xi} \right)^2$$

$$O_3 = \frac{1}{\Lambda^2} O_{\phi W} = \frac{1}{2} \frac{1}{\Lambda^2} g_w^2 v^6 h(\xi)^2 \left[ \frac{16}{\xi^2} f(\xi)^2 (1-f(\xi))^2 + \frac{8}{\xi^2} \left( \frac{df}{d\xi} \right)^2 \right]$$

$$O_4 = \frac{1}{\Lambda^2} O_{\phi}^{(1)} = \frac{1}{4} \frac{1}{\Lambda^2} g_w^2 v^6 h(\xi)^2 \left[ \left( \frac{dh}{d\xi} \right)^2 + \frac{2}{\xi^2} h(\xi)^2 (1-f(\xi))^2 \right]$$

$$O_5 = \frac{1}{\Lambda^2} O_{\phi}^{(3)} = \frac{1}{4} \frac{1}{\Lambda^2} g_w^2 v^6 h(\xi)^2 \left[ \left( \frac{dh}{d\xi} \right)^2 + \frac{1}{\xi^2} h(\xi)^2 (1-f(\xi))^2 \right]$$

$$O_6 = \frac{1}{\Lambda^2} O_w = \frac{24}{\xi^2} \frac{1}{\Lambda^2} g_w^2 v^6 \left( \frac{df}{d\xi} \right)^2 (1-2\xi \frac{df}{d\xi}) [4f(\xi)(1-f(\xi)) - 2\xi \frac{df}{d\xi}].$$

The energy functional is given by



$$E = \int d^3x \left\{ \frac{1}{4} F_{ij}^a F^{aij} + |D_i \phi|^2 + \lambda \left( \phi^2 - \frac{v^2}{2} \right)^2 \pm \frac{C_i}{\Lambda^2} O_i \right\},$$

where  $\Lambda$  is the energy for the new physics,  $C_i$  are positive constants which can be determined precisely if the dynamics at the scale  $\Lambda$  is known, and

$$F_{ij}^a = \partial_i W_j^a - \partial_j W_i^a + g_w \epsilon^{abc} W_i^b W_j^c$$

$$D_i \phi = \partial_i \phi - \frac{i}{2} g_w W_i^a \sigma^a \phi.$$

The sphaleron energy is obtained by minimizing the energy functional and thereby a set of two coupled non-linear differential equations of  $f$  and  $h$  are obtained. The field equations are given by

$O_1$ :

$$\xi^2 \frac{d^2 f}{d\xi^2} = 2f(1-f)(1-2f) - \frac{1}{4} \xi^2 h^2 (1-f)$$

$$\frac{d}{d\xi} \left[ \xi^2 \left( \frac{dh}{d\xi} \right) \right] = 2h(1-f)^2 + \frac{\lambda}{g_w} \xi^2 (h^2 - 1) + \left( \frac{v}{\Lambda} \right)^2 \frac{1}{4g_w^2} \xi^2 h (h^2 - 1)^2$$

$O_2$ :

$$\xi^2 \frac{d^2 f}{d\xi^2} = 2f(1-f)(1-2f) - \frac{1}{4} \xi^2 h^2 (1-f)$$

$$\frac{d}{d\xi} \left[ \xi^2 \left( 1 + \left( \frac{v}{\Lambda} \right)^2 h^2 \right) \left( \frac{dh}{d\xi} \right) \right] = 2h(1-f)^2 + \frac{\lambda}{g_w^2} \xi^2 h (h^2 - 1) + \left( \frac{v}{\Lambda} \right)^2 \xi^2 h \left( \frac{dh}{d\xi} \right)^2$$

$O_3$ :

$$\xi^2 \frac{d}{d\xi} \left[ \left( 1 + \left( \frac{v}{\Lambda} \right)^2 h^2 \right) \left( \frac{df}{d\xi} \right) \right] = 2 \left( 1 + \left( \frac{v}{\Lambda} \right)^2 h^2 \right) f(1-f)(1-2f) - \frac{1}{4} \xi^2 h^2 (1-f)$$

$$\frac{d}{d\xi} \left[ \xi^2 \left( \frac{dh}{d\xi} \right) \right] = 2h(1-f)^2 + \frac{\lambda}{g_w^2} \xi^2 (h^2 - 1)h + 8 \left( \frac{v}{\Lambda} \right)^2 h \left[ \left( \frac{df}{d\xi} \right)^2 + \frac{2}{\xi^2} f^2 (1-f)^2 \right],$$

$O_4$ :

$$\xi^2 \frac{d^2 f}{d\xi^2} = 2f(1-f)(1-2f) - \frac{1}{4} \left(1 + \frac{1}{2} \left(\frac{\nu}{\Lambda}\right)^2 h^2\right) h^2 (1-f)$$

$$\frac{d}{d\xi} \left[ \xi^2 \left(1 + \frac{1}{2} \left(\frac{\nu}{\Lambda}\right)^2 h^2\right) \left(\frac{dh}{d\xi}\right) \right] =$$

$$2 \left(1 + \frac{1}{2} \left(\frac{\nu}{\Lambda}\right)^2 h^2\right) h(1-f)^2 + \left(\frac{\lambda}{g_w^2}\right) \xi^2 h(h^2-1) + \left(\frac{\nu}{\Lambda}\right)^2 \left[\frac{1}{2} h \left(\frac{dh}{d\xi}\right) + h(h(1-f))^2\right]$$

$O_5$ :

$$\xi^2 \frac{d^2 f}{d\xi^2} = 2f(1-f)(1-2f) - \frac{1}{4} \left(1 + \frac{1}{2} \left(\frac{\nu}{\Lambda}\right)^2 h^2\right) h^2 (1-f)$$

$$\begin{aligned} \frac{d}{d\xi} \left[ \xi^2 \left(1 + \frac{1}{2} \left(\frac{\nu}{\Lambda}\right)^2 h^2\right) \left(\frac{dh}{d\xi}\right) \right] &= 2 \left(1 + \frac{1}{2} \left(\frac{\nu}{\Lambda}\right)^2 h^2\right) h(1-f)^2 + \left(\frac{\lambda}{g_w^2}\right) \xi^2 h(h^2-1) \\ &\quad + \left(\frac{\nu}{\Lambda}\right)^2 \left[\frac{1}{2} h \left(\frac{dh}{d\xi}\right) + \frac{1}{2} h(h(1-f))^2\right] \end{aligned}$$

$O_{6i}$ :

$$\begin{aligned} &\xi^2 \frac{d}{d\xi} \left[ \frac{df}{d\xi} + 3g_w \left(\frac{\nu}{\Lambda}\right)^2 \left( \frac{8f(1-f) \left(\frac{df}{d\xi} - 3\xi \left(\frac{df}{d\xi}\right)^2\right)}{\xi^2} - 6 \frac{\left(\frac{df}{d\xi}\right)^2}{\xi} + 16 \left(\frac{df}{d\xi}\right)^3 \right) \right] \\ &= 2f(1-f)(1-2f) - \frac{1}{4} \xi^2 h^2 (1-f) + 12g_w (1-2f) \left(\frac{\nu}{\Lambda}\right)^2 \left(\frac{df}{d\xi}\right)^2 (1-2\xi \left(\frac{df}{d\xi}\right)) \\ &\quad \frac{d}{d\xi} \left[ \xi^2 \left(\frac{dh}{d\xi}\right) \right] = 2h(1-f)^2 + \frac{\lambda}{g_w^2} \xi^2 (h^2-1). \end{aligned}$$

## 4.2. Numerical Solutions

In this section we will obtain the modifications of the sphaleron energy numerically. We will take  $C_i = 1$  and vary the high energy scale  $\Lambda$ . This is equivalent to introducing a varying  $C_i$  with a fixed energy scale  $\Lambda$ .

### 4.2.1. Perturbative Results

The contribution of the effective terms can be calculated by considering them as perturbations. We will use the Klinkhamer - Manton's ansatz [15] as the unperturbed gauge and Higgs fields. They are given by

$$f(\xi) = \frac{\xi^2}{\Xi} \quad , \text{ for } \xi \leq \Xi$$

$$\frac{\Xi}{(\Xi + 4)}$$

$$f(\xi) = 1 - \frac{4}{\Xi + 4} \exp\left[-\frac{1}{2}(\Xi - \xi)\right] \quad , \text{ for } \xi \geq \Xi$$

and

$$h(\xi) = \frac{\sigma\Omega + 1}{\sigma\Omega + 2} \frac{\xi}{\Omega} \quad , \text{ for } \xi \leq \Omega$$

$$h(\xi) = 1 - \frac{\Omega}{\sigma\Omega + 2} \frac{1}{\xi} \exp[\sigma(\Omega - \xi)] \quad , \text{ for } \xi \geq \Omega,$$

where  $\Xi$  and  $\Omega$  are determined by minimizing the energy functional in the absence of the effective terms for a given value of  $\frac{\lambda}{g_w^2}$ . This is given in the Table 1[15]. The corrections to

the sphaleron energy are given by

$$\Delta E_i = \pm \int d^3x \left\{ \frac{1}{\Lambda^2} O_i \right\}, \quad i = 1, 2, \dots, 6, \quad (1)$$

where we have put  $C_i$  to be 1. As expected, at the very low scale of  $\Lambda = 250$  GeV, the contributions can be large for some of the operators. For  $\Lambda \geq 1$  TeV, the corrections are

generally small. Since we treat the operators perturbatively, the bare Higgs mass is unaffected,  $M_H^2 = 2\lambda v^2 = 8\frac{\lambda}{g_W^2} M_W^2$  and the classical gauge and Higgs fields determined by  $f$  and  $h$  are not changed to the first order of the perturbation.

In Table 2, we list the contributions of six operators for several values of  $\frac{\lambda}{g_W^2}$  for the positive sign in (1) and  $\Lambda = 1$  TeV. When  $\Lambda$  changes, the contributions are simply rescaled by the ratio of the two  $\Lambda$ 's. For the negative sign in (1), all signs of values in Table 1 are reversed. Some of the salient feature of the perturbative calculations are given below:

(a)  $O_1$ : The contribution of  $O_1$  is negative and decreases in magnitude as the Higgs mass increases. The contribution vanishes for  $\frac{\lambda}{g_W^2} \geq 10^2$ . For a given value of  $\frac{\lambda}{g_W^2}$ , the contribution is proportional to the value of  $\Lambda^{-2}$ . At the very low scale of  $\Lambda = 250$  GeV, the contribution can be as large as - 7.7 TeV for small Higgs mass and reduces in magnitude to - 0.48 TeV for  $\Lambda = 1$  TeV. We found that for vanishing Higgs mass, i.e.,  $\lambda = 0$ , the expectation value of the operator diverges. This behavior, we think, reflects the particular form of the Klinkhamer - Manton's ansatz, rather than the actual behavior of the operator.

(b)  $O_2$ : The contribution is positive and decreases slowly as the Higgs mass increases. At the low scale of  $\Lambda = 250$  GeV, the contribution is about 1 TeV at zero Higgs mass, and at the scale of  $\Lambda = 1$  TeV, it is about 0.06 TeV. Therefore the contribution is in general very small for large scale  $\Lambda$ .

(c)  $O_3, O_4$ , and  $O_5$ : The behaviors of these three operators are similar. Their contributions increases as the Higgs mass increases. The contribution of  $O_3$  is about twice that of  $O_4$ , the contribution of  $O_5$  is about the same as  $O_4$  at zero Higgs mass, but it increases slower than that of  $O_4$ . Its values are about half of those of  $O_4$  for large Higgs mass. The change in

sphaleron energy is only a few percent in the case of  $\Lambda = 1$  TeV. But the contribution of  $O_3$  can be as large as 50% for  $\Lambda = 250$  GeV and large Higgs mass.

(d)  $O_6$ : The contribution of  $O_6$  changes sign from positive to negative near  $\frac{\lambda}{g_w^2} = 0.1$  when  $\frac{\lambda}{g_w^2}$  increases. But the magnitudes are all small with the maximal contribution occurring at  $\frac{\lambda}{g_w^2} = 10$ , while the value is - 0.22 TeV for  $\Lambda = 250$  GeV and - 0.014 TeV for  $\Lambda = 1$  TeV.

We conclude that the perturbative effect of the dimension 6 operators on the sphaleron energy is small for a new physics scale of  $\Lambda \geq 1$  TeV. However, if the scale is lower, the effect can be sizable.

#### 4.2.2. Non - Perturbative Results

Since we only investigate the vacuum sector, it would be interesting to incorporate the operators directly in the sphaleron energy equation, even though all 6 operators are non-renormalizable. The spherically symmetric forms of the gauge and Higgs fields are still valid. The boundary conditions are again valid for  $O_1, \dots, O_5$ . However, there are no consistent boundary conditions for  $f$  and  $h$  at  $r = 0$  with the inclusion of the operator  $O_6$ .

Therefore, we will not consider  $O_6$ . Note that  $O_6$  has a scaling behavior not contained in the original SU(2) - Higgs energy terms. We transform the integral from  $(0, \infty)$  to  $(0, 1)$  by using the transformation function

$$y = \ln\left(\frac{1+5(a+2)\cdot\xi}{1+5\cdot\xi}\right) / \ln(a+2), \quad a \equiv M_H/M_W.$$

Then the energy functional becomes

$$\int_0^1 E(f, f', h, h') dy$$

with the boundary conditions

$$y \rightarrow 0 ; f = h = 0$$

$$y \rightarrow 1 ; f = h = 1,$$

but these boundary conditions are not homogeneous. We decomposed  $f$  and  $h$  into two parts:

$$h = h_0 + \tilde{h}$$

$$f = f_0 + \tilde{f}$$

where  $f_0$  and  $h_0$  satisfies the original boundary condition and  $\tilde{f}$  and  $\tilde{h}$  satisfy the homogeneous boundary conditions i.e.,

$$\tilde{f}(0) = 0 \text{ and } \tilde{f}(1) = 0$$

$$\tilde{h}(0) = 0 \text{ and } \tilde{h}(1) = 0.$$

Here we approximate  $f_0$  and  $h_0$  using the known functions, for example, the exponential functions. As an approximation, we expanded the functions  $\tilde{f}$  and  $\tilde{h}$  in terms of the cubic functions  $B_\mu(y)$  i.e.,

$$\tilde{f} \equiv \sum_{\mu=1}^{I_{\max}} \alpha_{\mu,1} B_\mu(y)$$

$$\tilde{h} \equiv \sum_{\mu=1}^{I_{\max}} \alpha_{\mu,2} B_\mu(y),$$

where  $I_{\max}$  is the maximum number of spline functions ( $I_{\max} = 31$  in our case).  $B_\mu$ 's are cubic B-splines with two continuous derivatives a set of specially designed basis functions for approximation of functions[40]. We approximated the integral into summation by discretizing the interval (0,1) into 200 divisions. We used the minimizing sub-routine E04KCF-NAG Fortran Library to minimize the energy functional. E04KCF is a modified-Newton algorithm for finding a minimum of a function  $F(x_1, x_2, \dots, x_n)$ , subject to fixed

upper and lower bounds on the independent variables  $x_1, x_2, \dots, x_n$  when the first derivatives of  $F$  are available. We now describe the numerical results of the operators  $O_1$  to  $O_5$ .

The contributions are listed in Table 3. The contributions of  $O_2$  to  $O_5$  are similar to their perturbative parts. If the sign in front of the operator in (1) is chosen to be positive (negative), for  $O_2$  the nonperturbative energy is slightly below (above) those of the perturbative results for  $\Lambda = 250$  GeV, but slightly above (below) for  $\Lambda = 1$  TeV. For  $O_3$ , the nonperturbative results are slightly above those of the perturbative ones for all values of the Higgs mass and for the values of  $\Lambda$  investigated, when the positive sign in (1) is used. For the negative sign, the energy decreases slightly and the variation in energy is in general less than that for the positive sign. For  $O_4$  and  $O_5$ , the nonperturbative results are almost identical to those of the perturbative ones for either sign of the operators. The functions  $f$  and  $h$  do not change significantly away from the SU(2)-Higgs forms.

The behavior of the operator  $O_1$  is different and we will take  $\Lambda = 1$  TeV for illustration. For  $\frac{\lambda}{g_w^2} \geq 1$  (corresponding to the original Higgs mass greater than 226 GeV), the non-perturbative and perturbative results agree well for both signs. So do the functional forms of  $f$  and  $h$ . Therefore, the operator makes very little contribution to the sphaleron energy for large  $\frac{\lambda}{g_w^2}$ . However, for small  $\frac{\lambda}{g_w^2}$  the operator behaves quite differently. The sphaleron energy decreases abruptly at a critical  $\lambda$ ,  $\lambda_c$ , which depends on  $\Lambda$  and the sign of the operator in (1).

First consider the positive sign in (1). An abrupt change of energy takes place below  $\lambda_c$ , when  $\frac{\lambda_c}{g_w^2} \cong 0.008$  (corresponding to the original Higgs mass of 64 GeV). Below  $\lambda_c$  the sphaleron energy becomes large and negative, of the order of  $-10^6$  TeV and then increases slowly in magnitude and stays negative as  $\lambda$  further decreases. The change at  $\lambda_c$  is

discontinuous within the accuracy of our numerical analysis. We plot in Fig. 4.10 the sphaleron energy for  $\Lambda = 1$  TeV as a function of  $\frac{\lambda}{g_w^2}$  for  $O_1$ ,  $O_3$  and the original sphaleron.

$O_1$  and  $O_3$  give rise to the largest deviations among the 5 operators.

For the negative sign of (1), an even more dramatic behavior of the energy occurs. The sphaleron energy is slightly higher than the SU(2) - Higgs value for  $\lambda$  above the critical value  $\lambda'_c \cong 0.0012$ . Below this critical value the energy becomes negative and arbitrarily large.

### 4.3. Symmetry Restoration

We have seen from the preceding section that the energy of the sphaleron is in general exceedingly stable against the addition of non-renormalizable terms for not too small  $\Lambda$ .

The peculiar case is the addition of the operator  $O_1$ . The addition of  $O_1$  in the Lagrangian changes the Higgs potential :

$$V(\phi) = \lambda(\phi^2 - \frac{v^2}{2})^2 \pm \frac{1}{3\Lambda^2}(\phi^2 - \frac{v^2}{2})^3. \quad (2)$$

Let us consider the plus sign first. There is a critical  $\lambda_1 = \frac{1}{6} \frac{v^2}{\Lambda^2}$ , around which the potential changes its properties. For  $\lambda \geq \frac{3}{2} \lambda_1$ , the potential is in the broken phase and  $\phi = 0$  is a local maximum. For  $\lambda_1 < \lambda < \frac{3}{2} \lambda_1$ , the potential develops a local minimum at  $\phi = 0$  where it is a false vacuum and  $|\phi| = v/\sqrt{2}$  is still the true vacuum. However, for  $\lambda < \lambda_1$ , the potential becomes negative at  $\phi = 0$  where it is the true vacuum and is changed into a symmetric phase. For  $\Lambda = 1$  TeV,  $\lambda_1 \approx 0.01 > \lambda_c \approx 0.003$ . Therefore, it is reasonable to associate the abrupt change of the energy of the sphaleron with the change of the vacuum structure of the Higgs potential. In the symmetric phase, because the vacuum manifold is trivial, sphaleron type solutions are not expected to exist. There is a gap between the restoration of the



symmetry and the disappearance of the sphaleron solution. The physical meaning of this is not clear. We plot the change of the potential form for  $\Lambda = 1$  TeV as  $\lambda$  changes (Fig 4.1 to Fig. 4.7).

The case of the negative sign is quite different. The potential now is unstable, i.e.,  $V \rightarrow -\infty$  as  $|\phi| \rightarrow \infty$ . However, for  $\lambda > 0$ , the potential has a local maximum at  $\phi = 0$  and local minimum at  $|\phi| = v/\sqrt{2}$ . Our numerical result shows that as long as  $\lambda$  is not too small, the local minimum is able to support a sphaleron solution. The reason is that the boundary conditions of the sphaleron differential equation force the functions  $f$  and  $h$  to vanish at the origin and to become 1 at infinity. Therefore, the sphaleron is ordinarily sensitive only to the potential in the region  $|\phi| \leq v/\sqrt{2}$ , not for large  $|\phi|$ . Hence, even though the potential is unstable at large distance as it is in the present case, the sphaleron is not sensitive to it. However, for not too large  $\Lambda$  and for  $\lambda < \lambda_c$ , the minimum at  $|\phi| = v/\sqrt{2}$  is too shallow to support a positive energy sphaleron solution. We have also examined the cases of  $\Lambda \geq 2$  TeV, where we found that the sphaleron energy stays finite for all values of  $\lambda$ .

We have also examined the functional forms of  $f$  and  $h$  and found that, for  $\lambda > \lambda_c$  for the case of positive sign in (2), and for  $\lambda > \lambda'_c$  for the negative sign, they are smooth monotonic functions of  $\xi$ , very similar to those in the SU(2) - Higgs case. However, below these critical values, the function  $h$  becomes oscillatory near  $\xi = 0$ , while still satisfying the boundary conditions. In the case of the positive sign the oscillation is finite, while for the negative sign the oscillation becomes uncontrollably large. The function  $f$  is always a smooth function between 0 and 1. We plot  $f$  and  $h$  in Fig. 4.8 as a function of  $\xi$  for the positive sign in (2) and  $\Lambda = 1$  TeV just above the critical value  $\lambda_c$ . Both  $f$  and  $h$  are similar to their corresponding forms of the original sphaleron. Fig. 4.9 depicts the behavior of  $f$  and  $h$  just below  $\lambda_c$ , also for the positive sign in (2) and  $\Lambda = 1$  TeV. The large and negative energy of the sphaleron below  $\lambda_c$  is caused by the oscillation of  $h$  in the significant range of  $\xi$ .

Table 1. The optimal values of the scale parameters  $\Xi$  and  $\Omega$  for the ansatz.

$\frac{\lambda}{g_W^2}$	$\underline{\Omega}$	$\underline{\Xi}$
0	2.600	2.660
$10^{-3}$	2.520	2.450
$10^{-2}$	2.290	2.210
$10^{-1}$	1.900	1.650
1	1.250	1.150
10	0.620	0.820
$10^2$	0.220	0.740
$10^3$	0.070	0.730
$\infty$	0	0.728

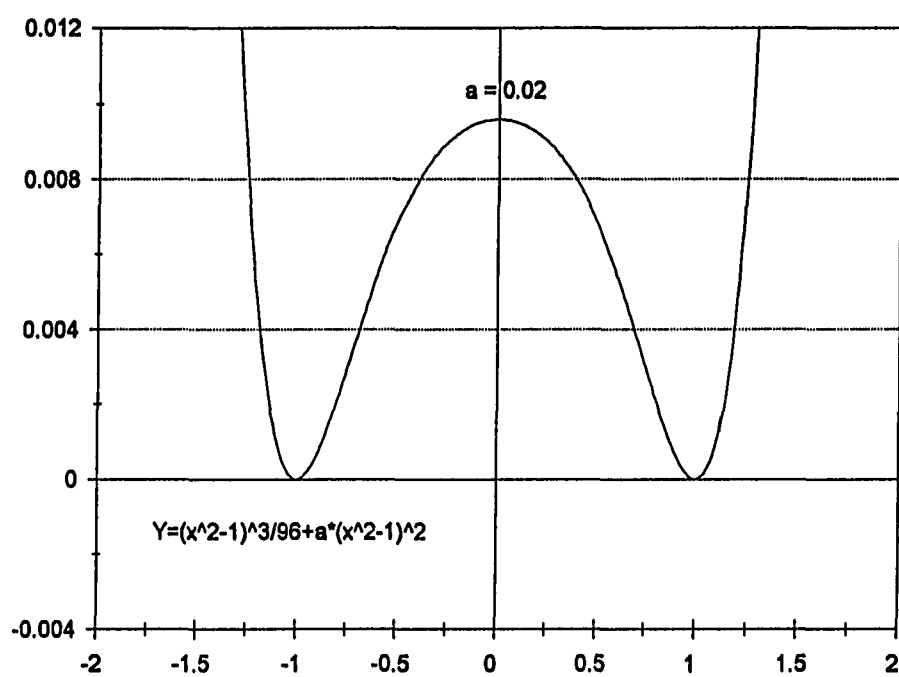


Fig. 4.1 Potential  $V(\phi)$  (positive sign) for  $\lambda > 3/2\lambda_1$

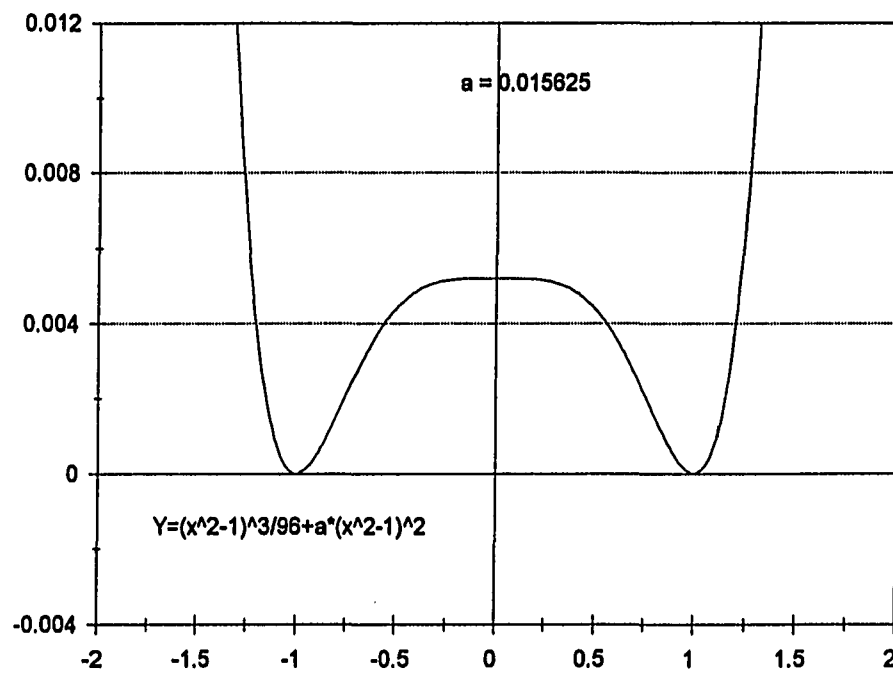


Fig. 4.2 Potential  $V(\phi)$  (positive sign) for  $\lambda = 3/2\lambda_1$

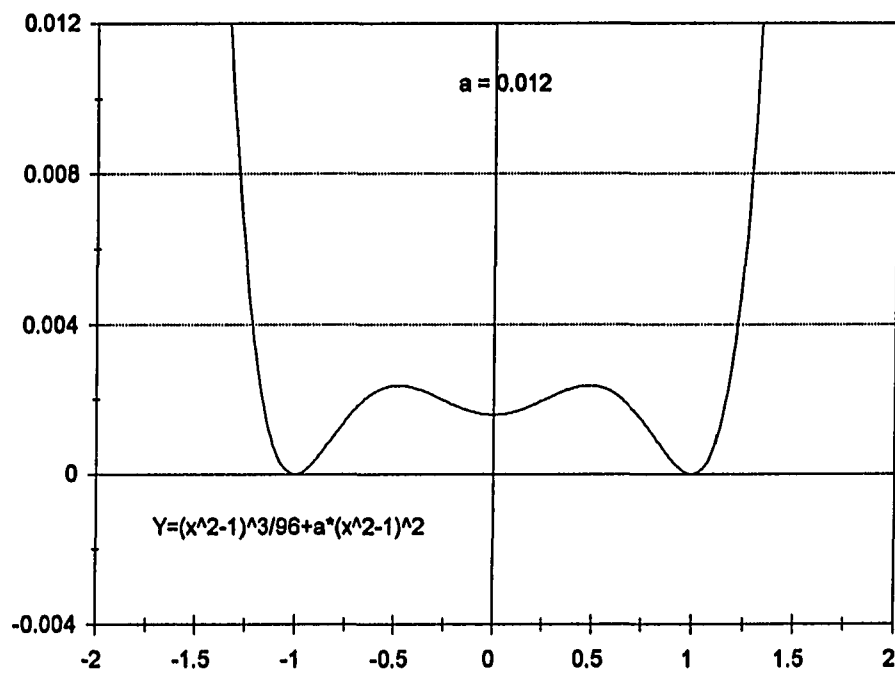


Fig. 4.3 Potential  $V(\phi)$  (positive sign) for  $\lambda_1 < \lambda < 3/2\lambda_1$

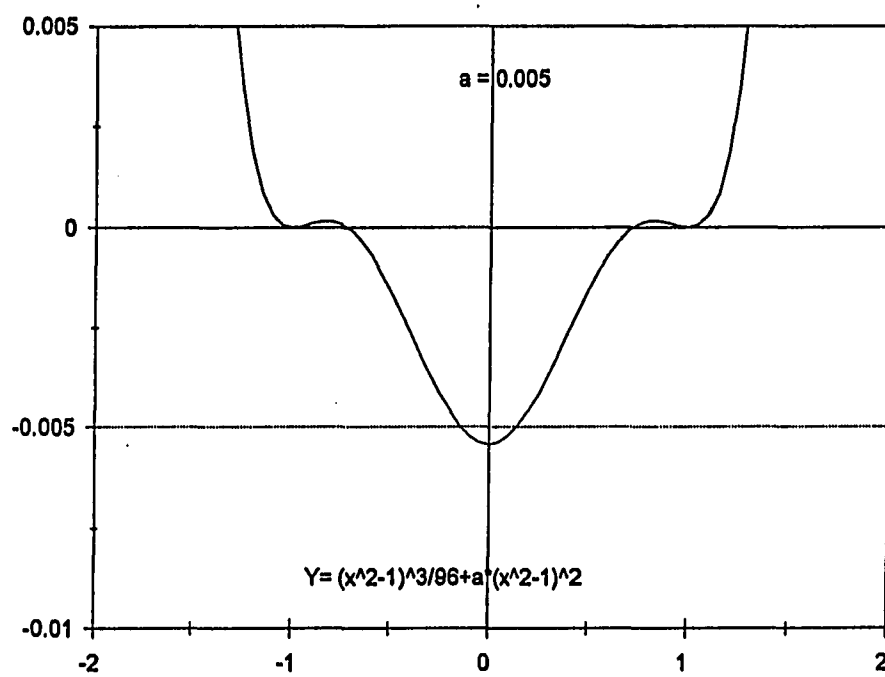


Fig. 4.4 Potential  $V(\phi)$  (positive sign) for  $\lambda < \lambda_1$

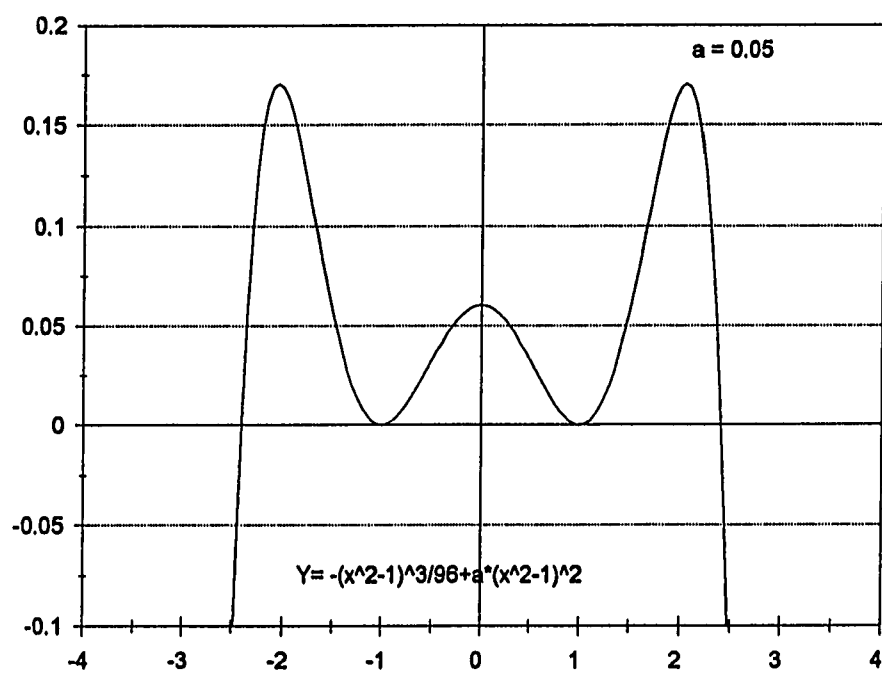


Fig. 4.5 Potential  $V(\phi)$  (negative sign) for  $\lambda > \lambda'_1$

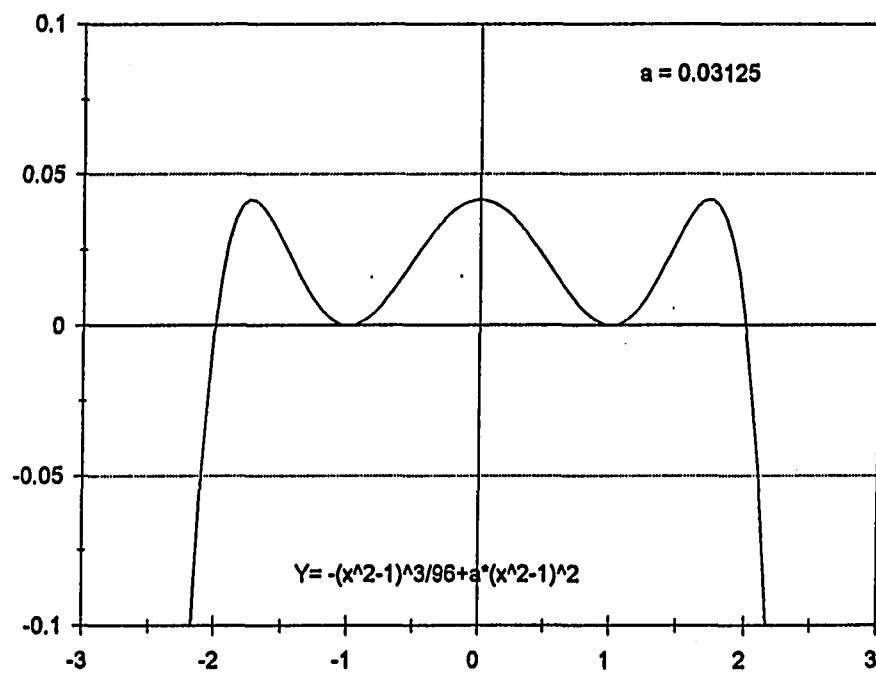


Fig. 4.6 Potential  $V(\phi)$  (negative sign) for  $\lambda'_c < \lambda < \lambda'_1$



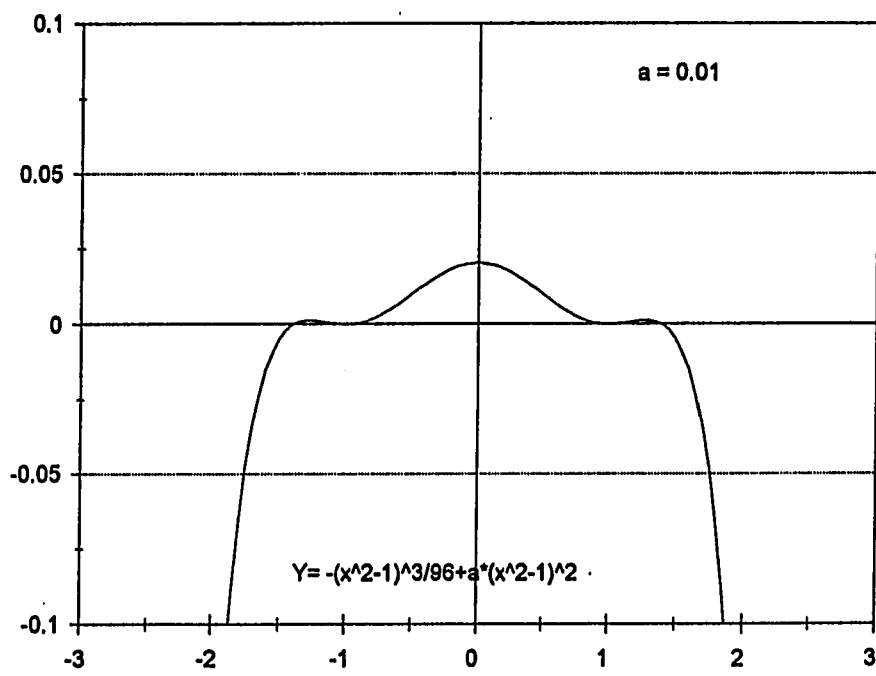


Fig. 4.7 Potential  $V(\phi)$  (negative sign) for  $\lambda'_c > \lambda$

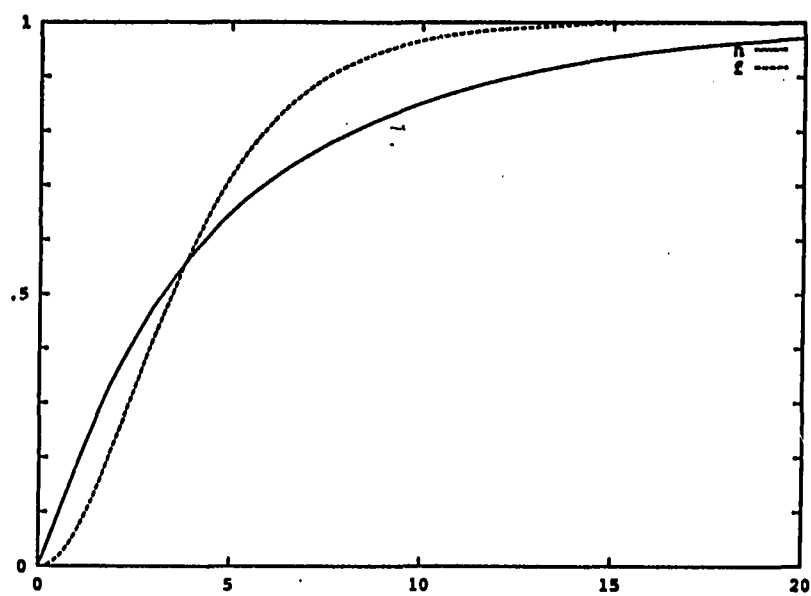


Fig. 4.8  $f$  and  $h$  functions before transition

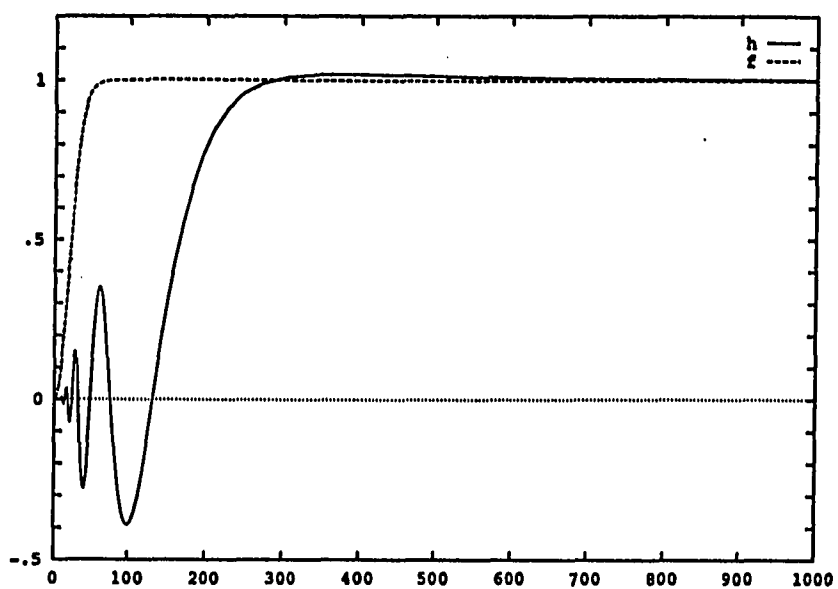


Fig. 4.9  $f$  and  $h$  function after the transition

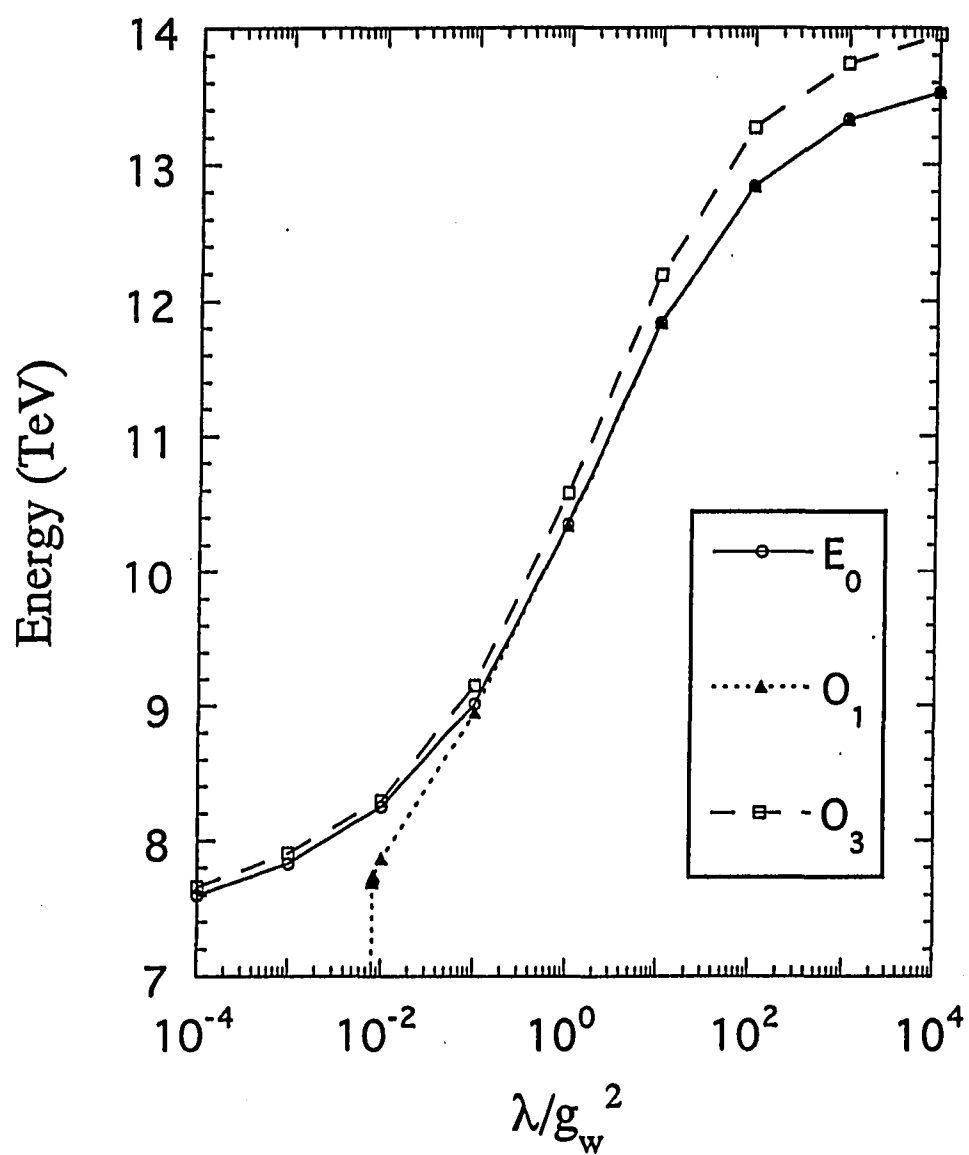


Fig. 4.10 Energy change of the operator  $O_1$  at the transition point.

Table 2. Perturbative contributions of the six operators at  $\Lambda = 1$  TeV.

(unit: TeV)

$\lambda/g_w^2$	$E_0$	$\Delta E_{o_1}$	$\Delta E_{o_2}$	$\Delta E_{o_3}$	$\Delta E_{o_4}$	$\Delta E_{o_5}$	$\Delta E_{o_6}$
0	7.597	$-\infty$	0.064	0.074	0.045	0.038	0.011
$10^{-3}$	7.832	-0.479	0.061	0.078	0.045	0.037	0.010
$10^{-2}$	8.205	-0.180	0.054	0.090	0.046	0.036	0.007
$10^{-1}$	9.010	-0.051	0.044	0.125	0.053	0.037	0
1	10.350	-0.009	0.030	0.221	0.081	0.047	-0.010
10	11.840	-0.001	0.015	0.339	0.118	0.062	-0.014
$10^2$	12.840	0	0.005	0.390	0.139	0.071	-0.013
$10^3$	13.330	0	0.002	0.411	0.170	0.085	-0.012
$\infty$	13.530	0	0	0.422	0.214	0.107	-0.012

Table 3. Non - perturbative contributions of the operators at  $\Lambda = 1$  TeV.

(unit: TeV)

$\lambda/g_w^2$	$E_0$	$O_1$	$O_2$	$O_3$	$O_4$	$O_5$
0	7.597	$-8.6 \cdot 10^5$	7.665	7.658	7.646	7.639
$10^{-3}$	7.832	$-7.8 \cdot 10^5$	7.894	7.904	7.881	7.872
$10^{-2}$	8.205	7.866	8.258	8.295	8.254	8.243
$10^{-1}$	9.010	8.950	9.050	9.148	9.065	9.048
1	10.350	10.340	10.370	10.580	10.420	10.390
10	11.840	11.840	11.850	12.190	11.950	11.900
$10^2$	12.840	12.840	12.870	13.270	13.030	12.950
$10^3$	13.330	13.320	13.330	13.740	13.520	13.420
$\infty$	13.530	13.530	13.530	13.950	13.740	13.640

## CHAPTER 5. CONCLUSIONS

1) The sphaleron energy changed at most a few percent, which has been shown by several other non-standard models[41], under the influence of the non-renormalizable dimension 6 operators except for one operator,  $O_1$ . The effect of the  $O_1$  operator clearly shows the character of the sphaleron solution, that is, that the solution exists only in the broken phase. Therefore, as long as we are in the broken phase, the calculation of baryon number violation rate based on the SU(2) - Higgs model remains valid[42].

2) Studies of the effects of the fermions, quantum corrections, and temperature on the sphaleron configuration are needed to understand the structure of the vacuum more thoroughly. Studies are also needed to higher order homotopy groups; the sphaleron solution is a result of the first non-trivial homotopy group.

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## APPENDIX

### A. Anomaly and Level Crossing.

Symmetries play crucial roles for the theory to be renormalizable and unitary. The Lagrangian must be chosen so that it fulfills the observed symmetry. There is no guarantee that the symmetry of a classical system can be elevated to a quantum symmetry, that is, the symmetry of the action. If the classical symmetry of a Lagrangian cannot be maintained in the process of quantization, the theory is said to have an Anomaly[43].

There are two types of anomalies; internal and external[44]. In the first case the symmetry we deal with is the gauge symmetry--the gauge symmetry of the classical action is violated at the quantum level. In other words, when quantum corrections (loop effects) are taken into account, the current with which gauge bosons interact ceases to be conserved. External anomalies also result in current non-conservation. In this case, however, the anomalous current is not connected with the gauge bosons and corresponds to "external" global symmetries of the classical action. Here we consider only the external anomaly. The main reason for the existence of this anomaly is due to the conflict between gauge invariance and the other invariance, for example, the chiral invariance in the extension of the theory to quantum level.

To gain a qualitative understanding of the situation we will examine the simple case of the axial anomaly. The main idea is to view the anomaly as arising from a flow of Landau level in the presence of external electric and magnetic fields. First we consider the massless electron moving in an electromagnetic field. We choose the applied uniform magnetic field  $B$  to be along the  $z$ -axis [45]:

$$A^2 = Bx, \quad A^\mu = 0 \quad \text{otherwise.}$$

Let us calculate the energy levels of the electron in this field. The equation of motion for the field  $\Psi$  is given by

$$(i\gamma \cdot \partial + e\gamma \cdot A)\Psi = 0$$

where the charge on the electron is  $-e$ ,  $e > 0$ . Hence the Hamiltonian is

$$H = \vec{\alpha} \cdot (\vec{p} + e\vec{A}) - eA^0 \quad \text{with } \vec{\alpha} = \gamma^0 \vec{\gamma}, \quad \vec{p} = -i \vec{\nabla}.$$

Write  $\Psi = \begin{pmatrix} \phi \\ \chi \end{pmatrix}$  and use the representation in which  $\vec{\alpha} = \begin{pmatrix} 0 & \vec{\sigma} \\ \vec{\sigma} & 0 \end{pmatrix}$  where  $\vec{\sigma}$  are the Pauli

matrices. The eigenvalue equation  $H|\rangle = E|\rangle$  then reduces to the pair of equations ( $A^0 = 0$ )

$$\vec{\sigma} \cdot (\vec{p} + e\vec{A})\chi = E\phi$$

$$\vec{\sigma} \cdot (\vec{p} + e\vec{A})\phi = E\chi.$$

The energy levels are given as

$$E^2 = p_z^2 + eB(2n+1+\alpha) \quad \text{where } n = 0, 1, 2, \dots$$

There is a continuous degeneracy in  $p_y$  and also a discrete degeneracy ; ( $n, \alpha = +1$ ) and ( $n+1, \alpha = -1$ ) have the same energy. However the levels ( $n = 0, \alpha = -1$ ) do not have this discrete degeneracy. The vacuum is defined by filling the negative energy Dirac sea and leaving the positive energy levels empty. Next we apply the uniform electric field adiabatically in the  $z$ -direction, antiparallel to magnetic field  $B$ . In the semiclassical approximation we write  $\dot{p}_z = -eE$ , which means that the levels move in the direction of the arrows in Fig. A.1.

For the ( $n = 0, \alpha = -1$ ) mode this leads to creation of right handed particles and anti-particles out of the vacuum leading to nonconservation of  $Q_5$ , the axial charge. This particle creation is the axial anomaly in the massless case. Since the electric field is turned on

adiabatically there is no particle creation in any of the other modes (all these modes have an energy gap).

These levels of course move as  $p_z$  changes but as each of these levels is twofold degenerate in  $\alpha$ , there is no flow of chirality associated with this motion. Let's consider the massive case here. The Hamiltonian given as

$$H = \vec{\alpha} \cdot (\vec{p} + e\vec{A}) + \gamma^0 m$$

leads to the following energy levels

$$E^2 = p_z^2 + m^2 + eB(2n+1+\alpha).$$

As before, there is a continuous degeneracy in  $p_y$  and a discrete degeneracy ;  $(n, \alpha = +1)$  and  $(n+1, \alpha = -1)$  have the same energy. But the level  $(n = 0, \alpha = -1)$  does not have this discrete degeneracy Fig. A.2. Note that all the levels now have an energy gap which means that for an adiabatically turned on electric field in the z-direction, there can be no particle production out of the vacuum. This is the effect of an explicit fermion mass. The point is that there is a zero energy level in the massless case but it is not in the massive case! Therefore the level crossing through the zero mode is possible in the massless case but it is impossible in the massive case when the external fields are applied adiabatically. The situation is different in electroweak theory where the fermion mass is dynamical--proportional to the Higgs fields--rather than being given explicitly, because the Higgs field also winds in an anomalous process[46]. Non-trivial winding of the Higgs field at the boundary of space - time implies that the Higgs field, and therefore the effective fermion mass, must be zero somewhere. This implies that the picture of level crossing out of the Dirac sea can be restored. So far we have only discussed anomalies of chiral symmetries. Chirality is violated but particle number is not; both a fermion on the line  $E = +p$  line and a hole on the  $E = -p$  line were produced at the same time. It is a general property of vectorially - coupled gauge

theories that only chiral symmetries can be anomalous. However, if the electric field had coupled only to positive chirality states, then only the  $E = + p$  line would have shifted, producing a fermion but no accompanying anti-fermion and hence violating fermion number: This is the situation with electroweak  $SU(2)$ , which couples only to left-handed particles. In Standard Model, the baryonic (leptonic) current is not conserved. The baryon number violating processes are believed to be mediated by sphaleron at the finite temperature.

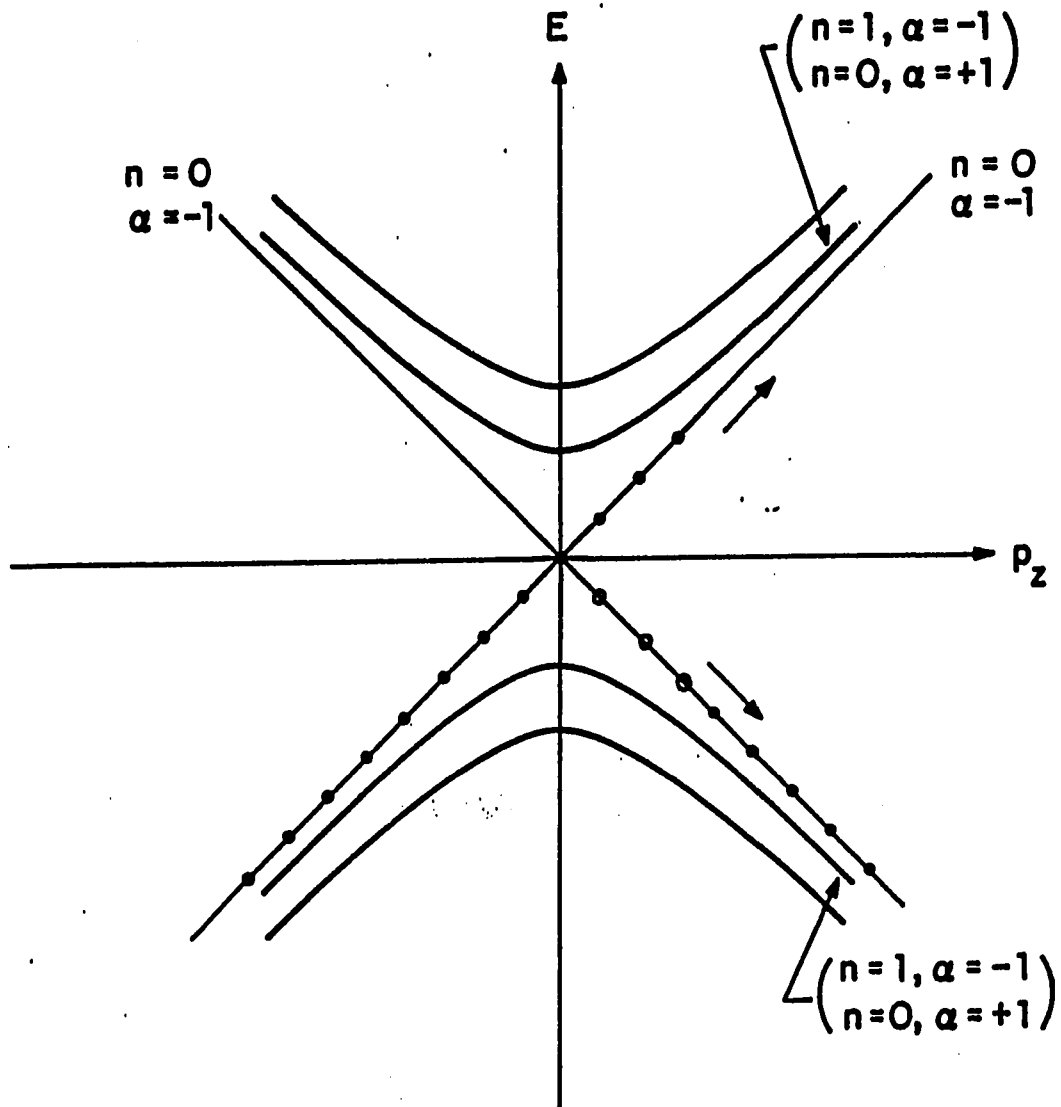


Fig. A.1 Level movement in massless case



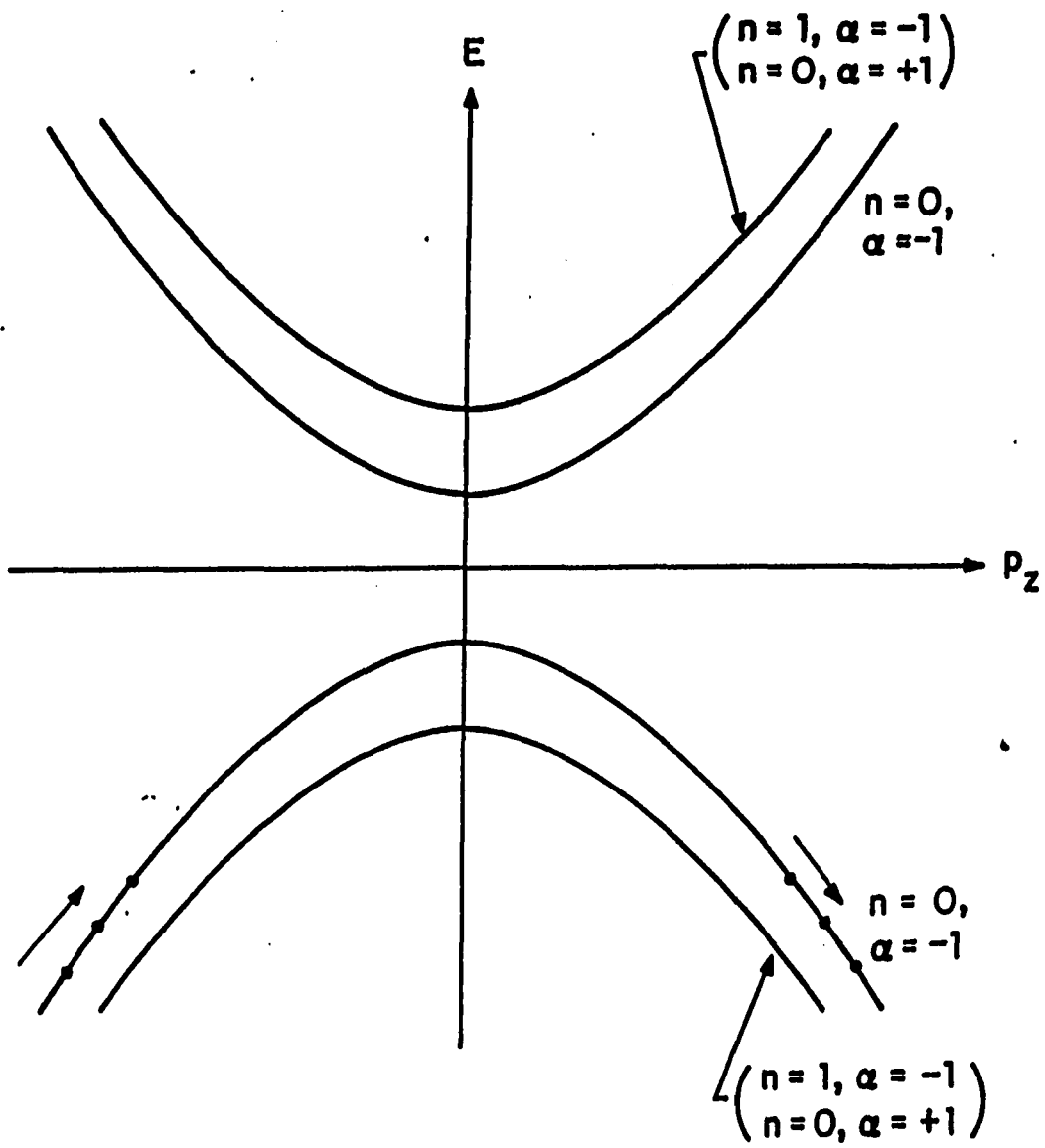


Fig. A.2 Level movement in massive case

### B. Fermion Zero Mode[47] in the Sphaleron Field

In order to connect the baryon creation with the level crossing picture, we have to prove that there actually exist fermion zero modes in the presence of sphaleron. The Lagrangian is given as

$$L = i \psi_L^\dagger (1_{2 \times 2} \cdot \partial^0 + \sigma \cdot D) \psi_L + \sum_{i=1}^2 i \psi_R^{(i)\dagger} (1_{2 \times 2} \cdot \partial_0 - \sigma \cdot \partial) \psi_R^{(i)} \\ - (h_1 \psi_L^\dagger \Phi^{Sp} \psi_R^{(1)} + h_2 \psi_L^\dagger \tilde{\Phi}^{Sp} \psi_R^{(2)} + H.C.)$$

in the Weyl representation. Then the field equations are

$$i(1 \cdot \partial_0 + \sigma \cdot D) \psi_L - h_1 \Phi^{Sp} \psi_R^{(1)} - h_2 \tilde{\Phi}^{Sp} \psi_R^{(2)} = 0$$

$$i(1 \cdot \partial_0 - \sigma \cdot \partial) \psi_R^{(1)} - h_1^* (\Phi^{+Sp} \psi_L) = 0$$

$$i(1 \cdot \partial_0 - \sigma \cdot \partial) \psi_R^{(2)} - h_2^* (\Phi^{+Sp} \psi_L) = 0.$$

For the zero energy  $\partial_0 \psi_L = \partial_0 \psi_R^{(i)} = 0$  We get

$$i\sigma^i (\partial_i - ig W_i^{Sp}) \psi_L - h_1 \Phi^{Sp} \psi_R^{(1)} - h_2 \tilde{\Phi}^{Sp} \psi_R^{(2)} = 0$$

$$-i\sigma^i \partial_i \psi_R^{(1)} - h_1^* (\Phi^{+Sp} \psi_L) = 0$$

$$-i\sigma^i \partial_i \psi_R^{(2)} - h_2^* (\Phi^{+Sp} \psi_L) = 0$$

Eq.(1) becomes

$$\sigma \cdot \partial \left\{ \frac{1}{h(\xi)r} (x \cdot \tau)^\dagger [(\sigma \cdot \partial \psi_L) - i \frac{f(\xi)}{r^2} (\sigma \times x) \cdot \tau \psi_L] \right\} \\ = \frac{v^2}{2} \frac{h(\xi)}{r} [(x \cdot \tau^T) M]^T \psi_L$$

where

$$\begin{aligned}
M^T &= |h_1|^2 \phi_0 \phi_0^+ + |h_2|^2 \tilde{\phi}_0 \tilde{\phi}_0^+ \\
\psi_R^{(1)} &= h_1^{-1} \frac{\sqrt{2}}{v} \frac{1}{h(\xi)r} \{ \phi_0^+ [(\underline{x} \cdot \underline{\tau})(\underline{\sigma} \cdot \partial \psi_L) - i \frac{f(\xi)}{r^2} \varepsilon_{ija} \sigma_i x_j (\underline{x} \cdot \underline{\tau}) \tau_a \psi_L] \} \\
\psi_R^{(2)} &= h_2^{-1} \frac{\sqrt{2}}{v} \frac{1}{h(\xi)r} \{ \tilde{\phi}_0^+ [(\underline{x} \cdot \underline{\tau})(\underline{\sigma} \cdot \partial \psi_L) - i \frac{f(\xi)}{r^2} \varepsilon_{ija} \sigma_i x_j (\underline{x} \cdot \underline{\tau}) \tau_a \psi_L] \}
\end{aligned}$$

let  $\psi_L = Q\tau^2$  and use  $\tau^2 \cdot \tau^T = -\tau \cdot \tau^2$ , then

$$\begin{aligned}
&\sigma \cdot \partial \left\{ \frac{1}{hr} [(\sigma \cdot \partial Q)(x \cdot \tau) \tau^2 + f \sigma^a Q \tau^a \tau^2 - \frac{f}{r^2} x^a (\sigma \cdot x) Q \tau^a \tau^2] \right\} \\
&= \frac{v^2}{2} \frac{h}{r} Q(x \cdot \tau) \tau^2 M.
\end{aligned}$$

From now on we don't need to distinguish between  $\sigma$  and  $\tau$ , so we will use  $\sigma$ . The final equation becomes

$$\begin{aligned}
&\sigma \cdot \partial \left\{ \frac{1}{hr} [(\sigma \cdot \partial Q)(x \cdot \sigma) + f \sigma^a Q \sigma^a - \frac{f}{r^2} (x \cdot \sigma) Q(x \cdot \sigma)] \right\} \\
&= \frac{v^2}{2} \frac{h}{r} Q(x \cdot \sigma) (\lambda_1 \cdot I - \lambda_2 \cdot \sigma^3)
\end{aligned}$$

where  $\lambda_1 = \frac{|h_1|^2 + |h_2|^2}{2}$  and  $\lambda_2 = \frac{|h_1|^2 - |h_2|^2}{2}$ .

We assume here  $h_1 = h_2$  for simplicity. In this case the solution is expected to be a spherically symmetric and it is enough to show the existence of the actual solution.

Let  $Q = g_0 \cdot \sigma_0 + \underline{g} \cdot \underline{\sigma}$  and multiply by  $(\underline{x} \cdot \underline{\sigma})$  to the left side and a few pages of algebra gives the final equation:

$$\begin{aligned}
\partial_r^2 g_0(r) + \left( 2 \frac{f(\xi)}{r} - \frac{h'(\xi)}{h(\xi)} \right) \partial_r g_0(r) + \frac{2f(\xi)}{r} \left( \frac{f'(\xi)}{f} - \frac{h'(\xi)}{h(\xi)} - \frac{1}{r} \right) g_0(r) &= \frac{v^2 h_1}{2} h(\xi)^2 g_0(r) \\
\underline{g} &= 0
\end{aligned}$$

Therefore, we have solution

$$\psi_L = \begin{pmatrix} g_0 & 0 \\ 0 & g_0 \end{pmatrix} \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} = \begin{pmatrix} 0 & -ig_0 \\ ig_0 & 0 \end{pmatrix} \quad \text{Q.E.D.}$$

### C. Anomalous Baryon Number Violation[48]

Classical vacuum configurations, i.e., fields with vanishing energy, are pure gauges. In the temporal gauge, , the classical vacua can be written in general as

$$\bar{W}(\vec{x}) = -\frac{i}{g} \nabla U U^{-1}, \quad \Phi_{vac} = U(\vec{x}) \Phi_0$$

where  $U(x)$  is a static  $SU(2)$  gauge transformation and  $\Phi_0 = (v/\sqrt{2})(0,1)^T$  denotes the vacuum expectation value. The allowed spatial gauge transformations  $U$  are mappings from  $R^3 \cup \{\infty\} \sim S^3$  onto  $SU(2) \sim S^3$ . Such mappings fall into homotopy classes, which may be classified by integer winding number  $n$ . Gauge transformations with different winding-numbers, called "large gauge transformations", are topologically inequivalent[49]. Physically this means that there is an energy barrier between topologically inequivalent vacua.

The winding number of the map  $U$

$$B(U) = \frac{1}{24\pi^2} \int d^3x \epsilon^{ijk} \text{Tr}[U^{-1} \partial_i U U^{-1} \partial_j U U^{-1} \partial_k U]$$

was considered to be connected to baryon number. Indeed  $B(U)$  is time independent and additive

$$B(U_1 U_2) = B(U_1) + B(U_2)$$

However, we see that  $B(U)$  is not gauge invariant under large gauge transformation, i.e.,  $U \rightarrow GU$ , where  $G$  has non-zero winding number. Thus  $B(U)$  can not be used to label physical states which are gauge invariant. There is another current which is gauge invariant:

$$\tilde{j}_B^\mu = \frac{1}{24\pi^2} \epsilon^{\mu\nu\alpha\beta} \text{Tr}[U^{-1} D_\nu U U^{-1} D_\alpha U U^{-1} D_\beta U + \frac{3}{2} i g U^{-1} F_{\nu\alpha} D_\beta U].$$

But  $\tilde{j}_B^\mu$  is not conserved, and we have

$$\partial_\mu \tilde{j}_B^\mu = \frac{-g^2}{16\pi^2} \text{Tr}(F_{\mu\nu} \tilde{F}^{\mu\nu}).$$

In fact it is impossible to choose a local current that is simultaneously gauge invariant and conserved. The charge associated with  $\tilde{j}_B^\mu$

$$\tilde{B} = \int d^3x \tilde{j}_B^0$$

is gauge invariant and can label physical states.

Since the divergence of the baryon number current is proportional to  $\text{Tr}(F\tilde{F})$ , whenever that quantity is sizable in a quantum process one may expect "topological" baryon number violation. Three mechanisms are possible. The first involves tunneling, and in a semi-classical description instantons are the dominant field configurations. But the tunneling rate, being exponentially small is negligible. The second is in the presence of monopole. It is possible because that the Pontryagin index (or winding number) of an 't Hooft-Polyakov monopole is also non-zero. The third possibility of the baryon number violation is in the presence of sphaleron. This process is intrinsically different from that of the process mediated by instanton based quantum tunneling. It is a classical process of passing over the energy barrier between the topologically distinct vacua through the sphaleron solution. The rate of this process shows no such severe suppression as shown in the instanton calculation. We will concentrate on the role of the instanton and sphaleron solutions in the anomalous baryon number violation process, both high energy and high temperature.

#### 1) High Energy.

't Hooft [50] showed the possibility, in the standard electroweak model, of the baryon number violation process in the context of zero- temperature , four dimensional instantons. But the rate is too small and it is practically ignorable. In the last decade or so, with the

discovery of sphaleron solution, people again started paying much attention to the possibility of the anomalous baryon number violation process in the high energy collider, like SSC. Of course, there is no consensus in this idea yet ; some people are pessimistic, some people are optimistic.

If the process is dominated through the sphaleron, the problem is the actual creation of the sphaleron state which is a rather large, coherent classical configuration from pp collider or  $e^+e^-$  collision. In perturbation theory, these processes are very suppressed since they originate at  $O(\alpha_W^{\frac{\pi}{\alpha_W}})$  and  $\alpha_W$  is a small parameter. Therefore the cross section roughly behaves

$$\begin{aligned}\sigma(2 \rightarrow \text{sphaleron}) &\sim \alpha_W^{\frac{\pi}{\alpha_W}} \\ &\sim \exp\left(-\frac{\pi}{\alpha_W} \log \frac{1}{\alpha_W}\right)\end{aligned}$$

which is also suppressed !

If the energy is sufficiently high, multiparticle production is allowed kinematically to occur. In order to produce  $n$  W's we need to have at least

$$\sqrt{s} > n M_W.$$

These kinematical arguments are not enough to make that happen. In order for these processes to become really important, it is necessary that the amplitudes for production not be rapidly damped, so that the natural phase space grows with energy, which favors multiparticle production. This happens when one can make a semiclassical approximation for the multiparticle Green's functions. Let's consider the scalar fields  $\phi$  to see the key idea. If we want to compute the physical amplitude for an  $n$ -leg process,  $A_n(p_1, p_2, \dots, p_n)$ , we have to consider the Green's function.

$$G_n(x_1, x_2, \dots, x_n) = \langle 0 | T(\phi(x_1)\phi(x_2)\dots\phi(x_n)) | 0 \rangle$$

Then the amplitude in question is given by

$$(2\pi)^4 \delta^4(\sum_i p_i) A_n(p_1, p_2, \dots, p_n) = \prod_i^n (p_i^2 + m_i^2) \int d^4 x_i e^{ip_i \cdot x_i} G_n(x_1, x_2, \dots, x_n) \Big|_{p_i^2 = m_i^2}.$$

We make here a semiclassical approximation for the Green's function  $G_n$ . The idea is to replace the quantum field  $\phi(x)$  by some classical field configuration. In general, such a classical field will depend not only on  $x$ , but also on some parameters which specify the location and other properties. Let's denote by  $\{\rho\}$  the collection of all parameters for the classical field, except its location,  $z$ . Then the semiclassical replacement intended for  $G_n$  replaces the quantum field  $\phi(x)$  by the classical field  $\phi_{cl}(x-z; \{\rho\})$ :

$$\phi(x) \rightarrow \phi_{cl}(x-z; \{\rho\})$$

and the Green's function becomes just a product of these classical fields, with some weighted integral, with weight  $e^{-S[\{\rho\}]}$ , over the parameters characterizing these fields.

$$G_n(x_1, x_2, \dots, x_n) = \int d^4 z d\{\rho\} e^{-S[\{\rho\}]} \prod_i \phi_{cl}(x-z; \{\rho\})$$

$$\prod_i \int d^4 x_i e^{ip_i \cdot x_i} G_n(x_1, x_2, \dots, x_n) = (2\pi)^4 \delta^4(\sum_i p_i) \int d\{\rho\} e^{-S[\{\rho\}]} \prod_i \int d^4 x_i e^{ip_i \cdot x_i} \phi_{cl}(x; \{\rho\})$$

One sees that in the semiclassical approximation  $A_n$  is just a point-like amplitude:

$$A_n(p_1, p_2, \dots, p_n) = \int d\{\rho\} e^{-S[\{\rho\}]} (Z[\{\rho\}])^n.$$

The phase space for producing  $n$  identical particles--assumed to be relativistic--grows like  $s^n$ :

$$\Phi_n = \frac{1}{n!} \prod_i \int \frac{d^3 p_i}{(2\pi)^3 2E_i} (2\pi)^4 \delta(\sqrt{s} - \sum_i E_i) \delta^3(\sum_i \vec{p}_i)$$

$$|\vec{p}_i| \gg m \rightarrow \left(\frac{1}{n!}\right)^3 \left[\frac{s}{16\pi^2}\right]^n \frac{1}{s}.$$

Typically, the semiclassical amplitudes  $A_n$  grows like  $n!$  for large  $n$ .

$$A_n = k \frac{n!}{\sigma^n}$$

where  $\sigma$  is a typical scale associated with the classical field  $\phi_{cl}$ .

Thus one obtains partial cross sections which are Poisson distributed

$$\sigma_n \cong \frac{k^2}{s} \frac{1}{n!} \left( \frac{s}{16\pi^2 \sigma^2} \right)^n$$

with a multiplicity which grows as a power of the energy and a total cross section which grows exponentially with the energy:

$$\sigma_{tot} = \sum_i \sigma_n \cong \frac{k^2}{s} \exp\left(\frac{s}{16\pi^2 \sigma^2}\right).$$

A. Ringwald [51] and O. Espinosa [52] suggested that at energies above the scale  $M_W / \alpha_W$  the nonperturbative phenomena may arise in the collision characterized by [53] :

(1) Copious production of W and Z bosons (as well as Higgs bosons) with a multiplicity growing rapidly with energy.

$$\langle n \rangle \sim \alpha_W^{1/3} (s / M_W^2)^{2/3}$$

(2) Violation of total fermion number by 6 units:

$$\Delta(B+L) = 6$$

We are interested in a computation of the amplitude of a (B+L)-violating quark-quark collision. For sufficiently high energy of incoming quarks, we must have in the final state, in addition to the minimum number of anti-fermions, a great number of W, Z and Higgs bosons;

$$q + q \rightarrow 7\bar{q} + 3\bar{l} + n_W W + n_Z Z + n_h \phi.$$

The total B-violating cross section  $\sigma_B$  obtained by Ringwald and Espinosa, and refined by McLerran et al. [54], is



$$\sigma_B(E) \sim \frac{1}{E^2} e^{-4\pi/\alpha_w} e^{cE^2/v^2} \quad (a)$$

where  $\alpha_w = g_w^2 / 4\pi \sim 1/30$ ,  $E$  is the total center-of-mass energy.  $v = 2M_w / g_w$  is the vacuum expectation value of the Higgs field, and  $c$  is a positive constant of order unity. Thus, at low energies,  $\sigma_B$  is controlled by the tiny tunneling factor  $\exp(-4\pi/\alpha_w)$ , with  $4\pi/\alpha_w$  the Euclidean action of the instanton. The second exponential, which is due to phase space, compensates the first at high energies,  $E \sim v/g_w \sim M_w / \alpha_w$  (sphaleron energy).

P. Arnold and M.P. Mattis [55], Yaffe [56] and Khlebnikov, Rubakov and Tinyakov [57] made corrections to this naive result (a). Consequently, Eq.(a) becomes

$$\sigma_B(E) \sim \frac{1}{E^2} \exp\left[-\frac{4\pi}{\alpha_w} f(E/E_{sph})\right]$$

where  $f(E/E_{sph}) \sim 1 - 9/8 (E/E_{sph})^{4/3} + 9/16 (E/E_{sph})^2 + o[(E/E_{sph})^{8/3}]$ .

Even if one cannot calculate  $f$ , it is possible that  $f$  remains positive for all values of  $E$ . In this case  $\sigma_B$  never violates unitarity, although if  $f < 1$ , the answer may be considerably larger than the 't Hooft estimate.

## 2) High Temperature

As temperature increases, the situation becomes quite different from that of the high energy. Dimopoulos and Susskind [58], Klinkhamer and Manton [14], and Kuzmin, Rubakov and Shaposhnikov (KRS) [42] have argued that at high temperatures baryon number violation is much larger than naive instanton calculation. The idea rests on the non-trivial vacuum structure of non-abelian gauge theories even at the weak coupling limit. The size of a sphaleron is of order  $1/M_w$ , which means its Fourier transform is dominated by momenta  $k \sim M_w$ , so the typical energy per quanta is of order  $M_w$ . The total energy of the sphaleron is roughly  $M_w / \alpha_w$ , and so the number of quanta in a sphaleron is  $N_{quanta} \sim 1/\alpha_w \sim 30$ . So, to cross the energy barrier from one vacuum to the other,  $\sim 30$  W's must come together, briefly

form a sphaleron, and fall apart. In a high-temperature plasma, finding 30 W's is no problem --there are as many W's present as any other species of particles. The number of quanta in a classical coherent state is Poisson distributed. The strength of events involving different numbers of particles should be roughly

$$P_n = \frac{\langle n \rangle^n}{n!} e^{-\langle n \rangle}.$$

For small  $n$ , this is actually suppressed by  $\exp(-\langle n \rangle)$ . The wave function constructed from the coherent-state representation shows Poisson distribution [59].

Let's consider a system with one degree of freedom, a particle in a periodic potential. Ignoring tunneling, the ground states of this system are harmonic oscillator ground states centered at each well. In WKB approximation, the tunneling amplitudes are exponentially small. But as one raises the temperature, the rate for transitions between the various ground states increases. For  $T \gg \omega$ , the system is classical. For  $T < V_0$ , where  $V_0$  is the barrier height, the flux across the top of the barrier is proportional to  $e^{-\beta V_0}$ . The rate of transition is just equal to the probability flux in one direction (from left to right) at the point  $x = 0$  [60]:

$$\text{Rate} = \langle \delta(x) \theta(v) v \rangle = \frac{\int dp dx \exp(-H/T) \delta(x) \theta(v) v}{\int dp dx \exp(-H/T)} = \frac{\varpi_0}{2\pi} \exp(-V_0/T).$$

For higher temperatures the rate is completely unsuppressed. In SU(2) - Higgs field case, the rate is given by

$$\Gamma = \frac{T^4 \varpi_-}{M_W(T)} \left( \frac{\alpha_W}{4\pi} \right)^4 N_{ir} N_{rot} \left( \frac{2M_W(T)}{\alpha_W T} \right)^7 \exp\left(-\frac{E_{sph}(T)}{T}\right) \kappa;$$

here  $N_{ir} \cong 26$ ,  $N_{rot} \cong 5.3 \times 10^3$  are the zero mode normalization,  $\kappa$  is the determinant of non-zero modes near the sphaleron [42] and  $E_{sph}(T) = 2M_W(T) / \alpha_W \cdot B(\lambda / \alpha_W)$  is the effective sphaleron energy accounting for the temperature evolution of the Higgs VEV [61].

From this argument, it is a reasonable assumption to assume that for high temperature below  $M_W / \alpha_W$ , the only suppression will be due to the Boltzmann factor associated with the top of the energy barrier,  $e^{-M_W/\alpha_W T}$ . At high temperatures, there should be no suppression at all. Therefore in theories with fermions, passage over the barrier should, through the anomaly, be accompanied by fermion emission. In the Standard Model, this means baryon (lepton) emission. Thus at temperatures near or above the Weinberg-Salam phase transition, the baryon number violation process should be unsuppressed.

PAPER II

T-ODD MUON POLARIZATION INDUCED BY FINAL STATE  
ELECTROMAGNETIC INTERACTION IN KAON DECAY

## CHAPTER 1. INTRODUCTION

The absolute character of CP invariance was believed until the  $K_L^0 \rightarrow \pi^+ \pi^-$  decay was discovered, and at first many were surprised by the CP breakdown. But nowadays people are surprised why the violation of CP is so weak. The breakdown of CP means that we have found that there are absolute differences between particles and antiparticles, between left and right, and that the microscopic world has its own arrow of time! Even though many work has been done in the last three decades or so, we still don't have a clear understanding of this subject yet.

The radiative decay  $K^+ \rightarrow \mu^+ \nu_\mu \gamma$  provides interesting information on the properties of hadronic weak currents. Since the final state of the decay consists of leptons and photons we can probe the properties of a hadronic weak current in the low-energy region without final-state strong interactions. In the framework of local quantum field theories, with Lorentz invariance and the usual spin-statistics connection, T violation implies CP violation (and vice versa), because of the CPT invariance of such theories[1]. Experimentally, only CP violation has been observed so far, and this only in kaons[2]. Empirically the violation of T symmetry has not been observed[3].

Outside this framework of local quantum field theories, there is no reason for the two symmetries to be linked. Therefore, it would be interesting to directly investigate T violation, rather than inferring it as a consequence of CP violation. Over the years, a number of processes have been suggested where one can look for either T or CP violation, e.g., kaon decays, B meson decays, rare  $Z^0$  decays, hyperon decay asymmetries, and electric dipole moments of the electron and the neutron[2]. Until now, CP violation has been observed only in a few decay modes of the kaon[3]. The standard model (SM), with at least three families of quarks and leptons, has a "natural" place for the T (or CP) violation, in terms of the mixing of

quarks; however, a more satisfactory and better understanding of this phenomenon is still lacking.

The weak decays in the sector of charged currents with  $\Delta S = 0, 1$  are described very well by the effective weak Hamiltonian

$$H_{eff}^{V-A} = \frac{G_F}{\sqrt{2}} [J_\mu^{+W}(x) J_\mu^W(x)],$$

where  $G_F$  is the Fermi coupling and  $J_\mu^W$  is the total charged weak current. From the V-A and universality weak interaction hypothesis, by considering only the effect of the three light quarks, the weak current  $J_\mu^W$  is given by

$$J_\mu^W(x) = J_\mu^h(x) + J_\mu^l(x),$$

where

$$J_\mu^h(x) = \cos \theta_c (V_\mu^{\Delta S=0} - A_\mu^{\Delta S=0}) + \sin \theta_c (V_\mu^{\Delta S=1} - A_\mu^{\Delta S=1})$$

$$J_\mu^l(x) = \sum_l [\bar{\psi}_{\nu_l} \gamma_\mu (1 - \gamma_5) \psi_l].$$

$J_\mu^l$  is the total weak lepton current and the sum is over the three lepton families,  $V_\mu^{\Delta S=0,1}$  and  $A_\mu^{\Delta S=0,1}$  are the vector and axial weak hadronic currents respectively, and  $\theta_c$  is the Cabibbo angle. All the experiments confirm the phenomenological predictions of the standard effective Hamiltonian. One signal of CP violation is the presence of a component of muon polarization normal to the decay plane in  $K_L^0 \rightarrow \pi^- \mu^+ \nu$  [4]. It is known that this normal polarization is small [5], so one must also consider the effects of electromagnetic final-state interactions, which induce a small normal polarization even in the absence of CP violation. The amount of electromagnetically induced normal polarization in the absence of CP violation was first calculated by Byers, MacDowell, and Yang [6]. A subsequent calculation of the same effect was performed by Okun and Khriplovich [7]. A more general result was obtained by Ginsberg and Smith [8]. Recently, the calculation is done on  $K^+ \rightarrow \pi^0 \mu^+ \nu$  [9] and  $K^+ \rightarrow \pi^+ \mu^+ \mu^-$  [10].

For the semileptonic decays of  $K_L^0$  [11], the relevant hadronic current part satisfies the relation

$$\frac{G_F}{\sqrt{2}} (2p_\pi^0 2p_K^0)^{1/2} \langle \pi^- | j_h^\mu(0) | K_L^0 \rangle = \frac{1}{2} (p_K + p_\pi)^\mu F_{L^+}^+ + \frac{1}{2} (p_K - p_\pi)^\mu F_{L^-}^+;$$

here  $F_{L^+}^+$  and  $F_{L^-}^+$  are the form factors. If these form factors are relatively real, then there is no CP violation in this process. So the imaginary part of the ratio

$$\xi^+ = \frac{F_{L^-}^+}{F_{L^+}^+}$$

is a measure of CP violation. For  $K_L^0 \rightarrow \pi^- \mu^+ \nu$  decay, the component of polarization normal to the decay plane is

$$\langle \vec{P}_+ \cdot \hat{n} \rangle = - \left[ \frac{d^2 \Gamma_+}{d(p_\pi \cdot p_\nu) d(p_\mu \cdot p_\nu)} \right]^{-1} \frac{M_K}{p_K^0} \frac{|F_{L^+}^+|^2}{(2\pi)^3} \frac{m_\mu^2}{M_K^3} \times |\vec{p}_\pi \times \vec{p}_\nu| \text{Im} \xi^+,$$

where  $\vec{p}_\pi$  and  $\vec{p}_\nu$  are the pion and neutrino three momenta in the muon rest frame,  $\hat{n} = (\vec{p}_\pi \times \vec{p}_\nu) / |\vec{p}_\pi \times \vec{p}_\nu|$ , and  $d^2 \Gamma / d(p_\pi \cdot p_\nu) d(p_\mu \cdot p_\nu)$  is the covariant differential decay rate. If CP were not violated, then  $\text{Im} \xi^+$  would be zero. However, a small amount of normal muon polarization would still be induced by an electromagnetic final state interaction. This effect is due to the interference between the tree diagram and the diagram of electromagnetic final state interaction between  $\pi^-$  and  $\mu^+$ . We will apply the same idea to the radiative decay of  $K^+$ :  $K^+ \rightarrow \mu^+ \nu_\mu \gamma$ . As far as we keep the form factors real there is no CP violation in this process. But from the interference between the tree diagram of  $K_{l2\gamma}$  and the diagram due to the final state electromagnetic interaction, we can generate the T-violating term from this process.

In Chapter 2, we will review briefly the discrete symmetry C, P, T and CPT theorem. In Chapter 3, a short discussion on the CP violation in the Kaon decays is given, and in Chapter 4, we analyze the radiative decay  $K^+ \rightarrow \mu^+ \nu_\mu \gamma$ . In Chapter 5, we calculate the transverse muon polarization  $P_\perp$  induced by the electromagnetic final state interaction in the radiative decay  $K^+ \rightarrow \mu^+ \nu_\mu \gamma$ . In Chapter 6, we present the conclusions.

## CHAPTER 2. C, P, T AND CPT THEOREM[12]

## 1) Charge Conjugation C

The charge conjugation operator C is defined so that a one particle state  $|f(\underline{p}, s)\rangle$  describing the particle f with momentum  $\underline{p}$  and spin projection s is transformed into the one particle state  $|\bar{f}(\underline{p}, s)\rangle$  describing the antiparticle  $\bar{f}$  with the same spin projection and the same momentum;

$$C|f(\underline{p}, s)\rangle = \eta_c |\bar{f}(\underline{p}, s)\rangle$$

where  $\eta_c$  is a phase factor. Under charge conjugation C, we have

$$C : r \rightarrow r$$

$$p \rightarrow p$$

$$e \rightarrow -e$$

$$s \rightarrow s.$$

## 2) Parity P

Under parity P, the space vector r is transformed into its opposite, -r

$$P|f(\underline{p}, s)\rangle = \eta_p |f(-\underline{p}, s)\rangle,$$

where  $\eta_p$  is a phase factor which is often called the intrinsic parity of the particle:

$$P : r \rightarrow -r$$

$$t \rightarrow t$$

$$p \rightarrow -p$$

$$E \rightarrow E$$

$$J \rightarrow J$$

$$\vec{A} \rightarrow -\vec{A}$$

$$\phi \rightarrow \phi$$

$$\vec{E} \rightarrow -\vec{E}$$

$$\vec{H} \rightarrow \vec{H}.$$



### 3) Time Reversal T

The operation T consists of reversing the clock:  $t$  is changed to  $-t$  while  $r$  is kept invariant. The operation of time reversal on a one particle state  $|f(\underline{p}, s)\rangle$  is described by an operator T such that:  $T|f(\underline{p}, s)\rangle = \eta_T^s |f(-\underline{p}, -s)\rangle$ , where  $\eta_T^s$  is a phase factor which depends on the initial spin  $s$ . Under the T operation:

$$T: r \rightarrow r$$

$$t \rightarrow -t$$

$$p \rightarrow -p$$

$$E \rightarrow E$$

$$J \rightarrow -J$$

$$\vec{A} \rightarrow -\vec{A}$$

$$\phi \rightarrow \phi$$

$$\vec{E} \rightarrow \vec{E}$$

$$\vec{H} \rightarrow -\vec{H}.$$

### 4) CPT Theorem[13]

In nature, some discrete symmetries are violated. Parity is maximally violated by the weak interactions, and the combination CP is violated in K-meson decays. However, there is a remarkable theorem that states that any quantum field theory is invariant under the combined operation of CPT. The theorem states that the Hamiltonian  $H$  is invariant under CPT:

$$(CPT)H(x)(CPT)^{-1} = H(x')$$

if the following two conditions are met:

1. The theory must be local, possess a Hermitian Lagrangian, and be invariant under proper Lorentz transformations.
2. The theory must be quantized with commutators for integral spin fields and quantized with anticommutators for half-integer spin fields (Spin-Statistics).

### CHAPTER 3. CP VIOLATION IN K - DECAYS[12,14]

#### 1) $K \rightarrow 2\pi$ decays

There are three two - pion decay modes for the kaon, namely,  $K^+ \rightarrow \pi^+ + \pi^0$ ,  $K^0 \rightarrow \pi^+ + \pi^-$  and  $K^0 \rightarrow \pi^0 + \pi^0$ , and similarly for two - pion decay modes of  $K^-$  and  $\bar{K}^0$ . Let's consider the CP and CPT properties of the  $2\pi$  decay modes of the neutral kaons. We note that for a boson - antiboson system the total wave function must be symmetric in space, spin, and charge conjugation C, i.e.,

$$C(-1)^l(-1)^s = 1$$

where  $l$  and  $s$  are the total angular momentum and total spin respectively. Thus, for the  $(\pi^+\pi^-)$  state,  $C = (-1)^l$  since the spin  $s$  of two pions  $s = 0$ . For the  $(\pi^0\pi^0)$  state,  $C$  must be positive; hence

$$CP|\pi^+\pi^- \rangle = (-1)^l |\pi^+\pi^- \rangle = |\pi^+\pi^- \rangle$$

$$CP|\pi^0\pi^0 \rangle = (-1)^l |\pi^0\pi^0 \rangle.$$

Thus CP of the  $(\pi^+\pi^-)$  state is always +1 while that of the  $(\pi^0\pi^0)$  state is also +1 since the spin of  $K^0$  is zero and therefore  $l$  must be zero ( we work in the rest frame of K). Now, in the presence of weak interactions,  $K^0 \rightarrow \bar{K}^0$  transitions are possible. Thus it is not  $K^0$  and  $\bar{K}^0$  that have definite lifetimes but some linear combinations of them, namely  $K_S^0$  and  $K_L^0$ , which are known as short-lived and long-lived components of  $K^0$  or  $\bar{K}^0$ . If all interactions are CP invariant, the states  $K_S^0$  and  $K_L^0$  are eigenstates of CP with eigenvalues +1 and -1 respectively. In weak interactions we must work with the states  $K_S^0$  and  $K_L^0$ . It then follows from the above discussion that if CP invariance is assumed only  $K_S^0$  (CP = +1) can decay into  $2\pi$  and the decay of  $K_L^0$  (CP = -1) into  $2\pi$  would be forbidden. Experimentally the ratio is given as

$$R = \left| \frac{\Gamma(K_L^0 \rightarrow \pi^+ + \pi^-)}{\Gamma(K_S^0 \rightarrow \pi^+ + \pi^-)} \right|^{1/2} \equiv 2 \times 10^{-3}.$$

Thus, although  $K_L^0 \rightarrow \pi^+ + \pi^-$  is seen and its most natural interpretation would be a CP violation in the weak interaction, nevertheless it is very small effect.

## 2) $K \rightarrow 3\pi$ decays

There are four three - pion decays for K decay, namely,

$$K^+ \rightarrow \pi^+ + \pi^+ + \pi^-$$

$$K^+ \rightarrow \pi^0 + \pi^0 + \pi^+$$

$$K^0 \rightarrow \pi^+ + \pi^- + \pi^0$$

$$K^0 \rightarrow \pi^0 + \pi^0 + \pi^0$$

and similarly for the three - pion modes of  $K^-$  and  $\bar{K}^0$ . If CPT invariance holds, we have  $\Gamma_{\bar{K}} = \Gamma_K$ . Now for  $K \rightarrow 3\pi$  decay, parity conservation forbids any strong electromagnetic transitions between a total  $J = 0$  three-pion state and a  $J = 0$  two-pion state; nor is either connected to leptonic final states except by the weak interaction. Thus, neglecting electromagnetism, two-pionic, three-pionic, and leptonic rates of K should be separately equal to the corresponding rates of  $\bar{K}$ .

### a) $3\pi$ modes of charged kaons

The above consideration show that CPT invariance implies

$$\begin{aligned} \Gamma(K^+ \rightarrow 3\pi) &\equiv \Gamma(K^+ \rightarrow \pi^+ + \pi^+ + \pi^-) + \Gamma(K^+ \rightarrow \pi^0 + \pi^0 + \pi^+) \\ &= \Gamma(K^- \rightarrow \pi^- + \pi^- + \pi^+) + \Gamma(K^- \rightarrow \pi^0 + \pi^0 + \pi^-) = \Gamma(K^- \rightarrow 3\pi). \end{aligned}$$

This is expected to hold to an accuracy of  $O(\alpha^2) \cong O(10^{-4})$  even if there is a large CP noninvariant amplitude in these decays. Thus any difference in the rates of  $K^+ \rightarrow 3\pi$  and  $K^- \rightarrow 3\pi$  would indicate violation of CPT.

### b) $3\pi$ modes of neutral kaons

$$K_i^0 \rightarrow 3\pi^0, \quad \pi^+ + \pi^- + \pi^0, \quad (i = L, S)$$

Both modes have been seen for the long-lived component  $K_L^0$ , but no adequate evidence exists for  $3\pi$  modes of the short-lived component  $K_S^0$ . For the final  $3\pi^0$  state,  $P = -1$ ,  $C = +1$ , and  $CP = -1$ . For the final  $(\pi^0\pi^+\pi^-)$  state,  $CP$  is given by

$$(-1)(-1)^{2L}(-1)^{l'} = (-1)^{2L+l'+1}$$

where  $L$  is the relative angular momentum of  $\pi^+$  and  $\pi^-$  and  $l'$  is the angular momentum of  $\pi^0$  with respect to the center of mass of the  $(\pi^+\pi^-)$ -system. Further, since the zero spin of  $K$  requires the final  $3\pi$  to be in a  $J = 0$  state, it follows that we must have  $L = l'$ . Thus  $CP$  of the  $(\pi^+\pi^-\pi^0)$  state is  $(-1)^{L+1}$ , which can be either  $+1$  or  $-1$  depending on the value of  $L$  or  $l'$ . From all these arguments, it follows that if  $CP$  invariance holds, then the decay  $K_S^0 \rightarrow 3\pi^0$  is completely forbidden and only  $K_L^0$  can decay into  $3\pi$ . This, in principle, provides a test of  $CP$  invariance in  $K^0 \rightarrow 3\pi^0$ .

CHAPTER 4. RADIATIVE DECAY  $K^+ \rightarrow \mu^+ \nu_\mu \gamma$  [15]

Let's start with the calculation of the amplitude for the process  $K^+(p) \rightarrow l^+(p_l) \nu_l(p_\nu) \gamma(q)$  where  $p$ ,  $p_l$ ,  $p_\nu$  and  $q$  are the corresponding four-momentum and the index  $l$  stands for a muon or an electron. The effective Lagrangian for the decay process can be written as follows:

$$L_{eff} = (D_\mu K^+)(D^\mu K^-) + \frac{ieG_F}{\sqrt{2}} \sin \theta_c m_l (D_\mu K^+) \bar{\nu} \gamma^\mu (1 - \gamma_5) l + i \bar{l} \not{D} l + \\ - \frac{ieG_F}{\sqrt{2}} \sin \theta_c \{ D_\mu K^+ \cdot \bar{\nu} \gamma_\nu (1 - \gamma_5) l \cdot F^{\mu\nu} + \epsilon^{\mu\nu\alpha\beta} D_\mu K^+ \cdot \bar{\nu} \gamma_\nu (1 - \gamma_5) l \cdot F_{\alpha\beta} \} + \text{h.c.}$$

The Feynman diagrams at tree level are given in Fig. 1. The first two diagrams 1a, 1b are the so - called inner bremsstrahlung (IB) diagrams, because the photon  $\gamma$  is emitted by the external lines. The diagram in Fig. 1c is the so-called "structure-dependent" (SD) diagram, because the photon is emitted by an intermediate hadronic state (different from the K) and its matrix element will depend on form factors related to the low-energy effect of the strong interactions. The form factors which appear in the SD diagram are well measured and are constant up to 1-loop contribution. Assuming the standard V-A theory, the matrix element  $M_1$  for the diagram of Fig. 1a is given by

$$M_1 = \frac{ieG_F}{\sqrt{2}} \sin \theta_c f_K \epsilon^{\mu*} p^\beta ((\bar{u}(p_\nu) \gamma_\beta (1 - \gamma_5) \frac{q + p_l - m_l}{(q + p_l)^2 - m_l^2} \gamma_\mu \nu(p_l))$$

where  $\epsilon_\mu$  is the photon polarization vector,  $m_l$  is the mass of the lepton  $l$  and  $f_K$  is the K decay constant defined as

$$\langle 0 | A_4^\mu - i A_5^\mu | K^+(p) \rangle = i f_K p^\mu.$$

The indices 4, 5 are the usual indices of flavour  $SU(3)_F$  axial current. The matrix element  $M_2$  obtained by summing the diagram 1b and 1c is the following:

$$M_2 = -\frac{ieG_F}{\sqrt{2}} \sin \theta_c \varepsilon^{\mu*} [\bar{u}(p_\nu) \gamma^\beta (1 - \gamma_5) v(p_l)] [V_{\mu\beta}(p, q) - A_{\mu\beta}(p, q)]$$

where the tensors  $V_{\mu\beta}(p, q)$  and  $A_{\mu\beta}(p, q)$  are defined through

$$I^{\mu\beta}(p, q) = \int d^4x e^{iqx} \langle 0 | T V_{em}^\mu(x) [I_4^\beta(0) - iI_5^\beta(0)] | K^+(p) \rangle, \quad I = V, A$$

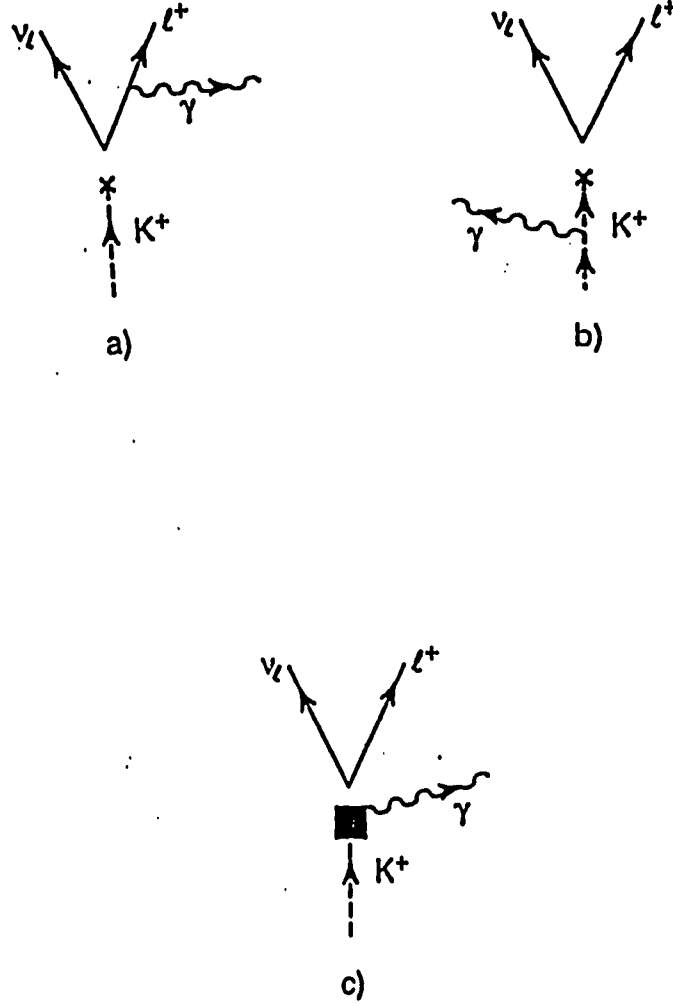


Fig. 6.1 Feynman diagram

and  $V_{em}^\mu$  is the hadronic electromagnetic current. In the quark model, the currents used in the test are given by

$$V_4^\mu - iV_5^\mu = \bar{\psi}_s \gamma^\mu \psi_u, \quad A_4^\mu - iA_5^\mu = \bar{\psi}_s \gamma^\mu \gamma_5 \psi_u, \quad V_{em}^\mu = \frac{2}{3} \bar{\psi}_u \gamma^\mu \psi_u - \frac{1}{3} \bar{\psi}_d \gamma^\mu \psi_d - \frac{1}{3} \bar{\psi}_s \gamma^\mu \psi_s.$$

By the electromagnetic gauge invariance, the tensor  $I_{\mu\nu}$  satisfies the following identities:

$$q^\mu A_{\mu\beta} = -f_K p_\beta \quad \text{and} \quad q^\mu V_{\mu\beta} = 0.$$

Using the Lorentz covariance and the electromagnetic gauge invariance determines the general forms of  $V_{\mu\nu}$  and  $A_{\mu\nu}$  as follows:

$$V_{\mu\nu} = i \frac{V}{M_K} \varepsilon^{\mu\nu\alpha\beta} q_\alpha p_\beta, \quad A_{\mu\nu} = (pq) \frac{A}{M_K} (g_{\mu\nu} - \frac{p_\mu q_\nu}{(pq)}) - f_K (g_{\mu\nu} + \frac{p_\mu q_\nu}{(pq)})$$

where  $M_K$  is the mass of the K, the four-momentum  $Q_\mu = p_\mu - q_\mu$  and A, V are the axial, vector form factor respectively. For CP invariance V and A are real:

$$V + A = -0.137 \quad \text{and} \quad V - A = -0.052.$$

We have the total matrix element  $M = M_{IB} + M_{SD}$

$$M_{IB} = \frac{ieG_F}{\sqrt{2}} \sin \theta_c m_l f_K \varepsilon_\mu^*(q) [\bar{u}(p_\nu)(1 + \gamma_5) (\frac{p_\mu}{(pq)} - \frac{q \gamma^\mu + 2p_l^\mu}{2(p_l q)}) v(p_l)]$$

$$M_{SD} = -\frac{ieG_F}{\sqrt{2}} \sin \theta_c \varepsilon_\mu^* H^{\mu\nu} L_\nu$$

$$\text{where } H^{\mu\nu} \equiv (pq) \frac{A}{M_K} (-g_{\mu\nu} + \frac{p_\mu q_\nu}{(pq)}) + i \varepsilon_{\mu\nu\alpha\beta} \frac{A}{M_K} q^\alpha p^\beta, \quad L_\nu \equiv \bar{u}(p_\nu) \gamma_\mu (1 - \gamma_5) v(p_l).$$

After calculating the square modulus  $|\bar{M}|^2$  summed over polarizations and integrating over the phase space, we obtain for the differential decay width  $\Gamma^l$  in the leptonic mode  $l$  the following result:

$$\frac{d^2 \Gamma^l}{dx d\lambda} = \frac{M_K}{256\pi^3} |\bar{M}|^2 = A_{SD} \rho(x, \lambda), \quad \text{where } A_{SD} = \frac{\alpha}{32\pi^2} G_F^2 M_K^3 \sin^2 \theta_c.$$

The kinematical variables  $x$  and  $\lambda$  are defined as  $x = 2pq / M_K^2$  and  $\lambda = (x + y - 1 - r_l) / x$  with  $y = 2pp_l / M_K^2$ . In the K frame the variables  $x$  and  $\lambda$  are given by

$$x = \frac{2E_\gamma}{M_K}, \quad \lambda = \frac{E_l}{M_K} \left( 1 - \sqrt{1 - \frac{r_l M_K^2}{E_l^2}} \cos \theta_\gamma \right)$$

where  $E_\gamma$  and  $E_l$  are the energy of the photon and lepton  $l$  respectively and  $\theta_\gamma$  is the angle between their three momenta. Here  $r_l \equiv (m_l / M_K)^2$  and  $\rho(x, \lambda)$  is given by

$$\rho(x, \lambda) = \rho_{IB}(x, \lambda) + \rho_{SD}(x, \lambda) + \rho_{IBSD}(x, \lambda)$$

where

$$\rho_{IB}(x, \lambda) = 2r_l \left( \frac{f_K}{M_K} \right) f_{IB}(x, \lambda), \quad \rho_{SD}(x, \lambda) = \frac{1}{2} [(V+A)^2 f_{SD^+}(x, \lambda) + (V-A)^2 f_{SD^-}(x, \lambda)]$$

$$\rho_{IBSD}(x, \lambda) = 2r_l \frac{f_K}{M_K} [(V+A) f_+(x, \lambda) + (V-A) f_-(x, \lambda)]$$

and

$$f_{IB}(x, \lambda) = \frac{1-\lambda}{\lambda x} (x^2 + 2(1-x)(1-r_l) - \frac{2r_l(1-r_l)}{\lambda})$$

$$f_{SD^+}(x, \lambda) = x^2 \lambda [(\lambda x + r_l)(1-x) - r_l], \quad f_{SD^-}(x, \lambda) = x^2 (1-\lambda) [(x-1)[r_l + x(\lambda-1)] + r_l]$$

$$f_+(x, \lambda) = \frac{1-\lambda}{\lambda} [(x-1)(x\lambda + r_l) + r_l], \quad f_-(x, \lambda) = \frac{1-\lambda}{\lambda} [x^2 + (1-x)(x\lambda + r_l) + r_l].$$



## CHAPTER 5. FINAL STATE INTERACTION AND T - ODD MUON POLARIZATION

We will consider the effect of the electromagnetic final state interaction to the radiative decay  $K_{l2\gamma}$  to 1 - loop. We will denote the tree level  $K_{l2\gamma}$  amplitude as follows

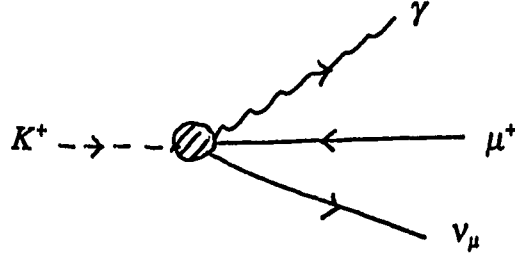


Fig. 6.2 Tree diagram of  $K_{l2\gamma}: M_0$

There are two possible final state interaction graphs due to electromagnetic interaction

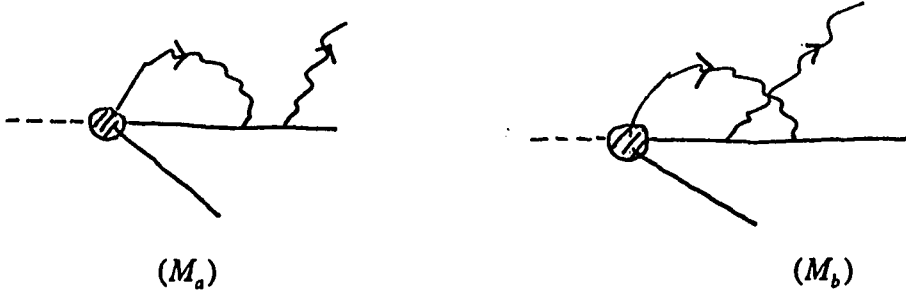


Fig. 6.3 Two possible final-state interaction diagrams

Then the amplitude  $M$  is given by

$$M = M_0 + M_a + M_b$$

$$|M|^2 = M^* M = |M_0|^2 + 2\text{Re}\{M_0^* (M_a + M_b)\} + \dots$$

Therefore, the differential decay width of the process is given by

$$\frac{d^2\Gamma}{dx dy} = \frac{M_K}{256\pi^3} |M|^2.$$

The Polarization  $P_1$  is defined as the ratio of the coefficient of the quantity  $\varepsilon^{\mu\nu\alpha\beta} p^\mu s^\nu q^\alpha l^\beta$  and the tree decay width  $A_{SD}\rho(x,y)$ , i.e.,

$$P_{\perp} = \frac{(\text{Coefficient}) \cdot \epsilon^{\mu\nu\alpha\beta} p^{\mu} s^{\nu} q^{\alpha} l^{\beta}}{A_{SD} \rho(x, y)}.$$

We are interested in the muon polarization called transverse polarization  $P_{\perp}$  which is related to the T - odd triple correlation which is given in the rest frame of the Kaon

$$\epsilon^{\mu\nu\alpha\beta} p^{\mu} s^{\nu} q^{\alpha} l^{\beta} = M_K \cdot \vec{S}_{\mu} \bullet (\vec{P}_{\gamma} \times \vec{P}_{\mu})$$

where  $\vec{S}_{\mu}$  and  $\vec{P}_{\mu(\gamma)}$  are the muon spin vector and the muon (photon) momentum respectively.

This triple correlation appears only from the imaginary part of the diagram  $M_a$  and  $M_b$  in the trace calculation. From the optical theorem, it is well known that the total cross section is connected to the imaginary part of the sub - processes. Technically, the calculation of the imaginary part is equivalent to cutting the loop and putting every particle which is being cut onto mass shell, i.e., physical particle[16]. The cross section for a  $2 \rightarrow n$  reaction is given by

$$\sigma_{2 \rightarrow n} = \frac{1}{2\lambda(s, p_a, p_b)} \int \prod_{j=1}^n \left[ \frac{dk_j}{(2\pi)^3} \delta(k_j^2 - m_j^2) \right] (2\pi)^4 \delta\left(\sum_{j=1}^n k_j - p_a - p_b\right) |T_{2 \rightarrow n}|^2$$

and the total cross section is given by

$$\sigma_T = \sum_n \sigma_n.$$

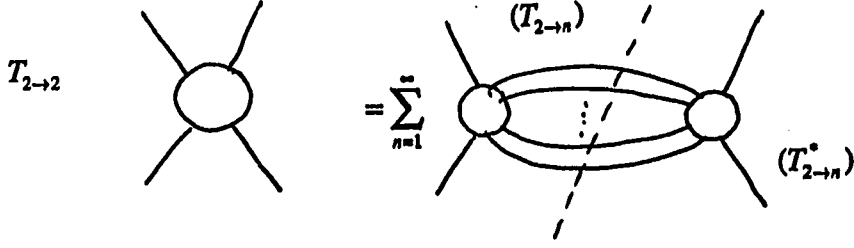
From the optical theorem

$$\sigma_T = \frac{1}{\lambda(s, p_a, p_b)} \text{Im} T_{2 \rightarrow 2}.$$

Then

$$\text{Im} T_{2 \rightarrow 2} = \frac{1}{2} \sum_n \int \prod_{j=1}^n \left[ \frac{dk_j}{(2\pi)^3} \delta(k_j^2 - m_j^2) \right] (2\pi)^4 \delta\left(\sum_{j=1}^n k_j - p_a - p_b\right) |T_{2 \rightarrow n}|^2.$$

Let us rewrite the  $2 \rightarrow 2$  elastic scattering amplitude  $T_{2 \rightarrow 2}$



$$= \sum_{n=1}^{\infty} \int \prod_{j=1}^n \frac{d^4 k_j}{(2\pi)^4} \frac{i}{k_j^2 - m_j^2 + i\epsilon} (2\pi)^4 \delta(\sum_j k_j - p_a - p_b) (T_{2 \to n})(T_{2 \to n}^*).$$

Let us replace all propagators by their imaginary part multiplied by 2 i.e.,

$$\frac{i}{k_j^2 - m_j^2 + i\epsilon} \rightarrow 2i(-i\pi\delta(k_j^2 - m_j^2)) = 2\pi\delta(k_j^2 - m_j^2).$$

Then

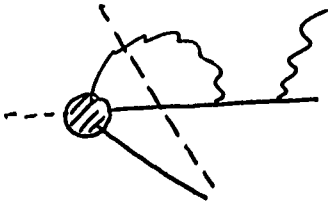
$$\text{Im} T_{2 \to 2} \sim \sum_n \int \prod_{j=1}^n \left[ \frac{dk_j}{(2\pi)^3} \delta(k_j^2 - m_j^2) \right] (2\pi)^4 \delta(\sum_{j=1}^n k_j - p_a - p_b) |T_{2 \to n}|^2.$$

Therefore, the general rule to cut the internal lines into two parts of physical processes is to replace each propagator with its imaginary part, i.e.,

$$\frac{1}{p_\gamma^2 + i\epsilon} \rightarrow -2\pi i \delta(p_\gamma^2) \text{ for photon and } \frac{1}{p_l^2 - m_l^2 + i\epsilon} \rightarrow -2\pi i \delta(p_l^2 - m_l^2) \text{ for fermion,}$$

and divide the whole expression by 2. So the expression of the cut diagram is given as follows,

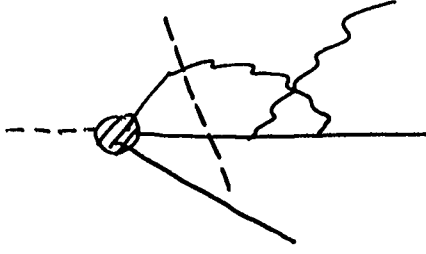
(a)



$$M_a = \bar{u}(k) \int \frac{d^4 p_\gamma}{(2\pi)^4} \frac{d^4 p_l}{(2\pi)^4} \left( \frac{i}{p_\gamma^2 + i\epsilon} \right) \Gamma \left( \frac{i}{-p_l - m_l + i\epsilon} \right)$$

$$\begin{aligned}
& (-ie\tilde{\epsilon})\left(\frac{i}{-l-q-m_l+i\epsilon}\right)(ie\tilde{\epsilon}^*)v(l)(2\pi)^4\delta(P_1-p_\gamma-p_l) \\
& = \frac{e^2}{2(2\pi)^4}\bar{u}(k)\int d^4p_\gamma d^4p_l(2\pi)^2\delta(P_1-p_\gamma-p_l)\delta(p_\gamma^2) \\
& \quad \delta(p_l^2-m_l^2)\Gamma(-p_l+m_l)\tilde{\epsilon}\left(\frac{i}{-l-q-m_l+i\epsilon}\right)\tilde{\epsilon}^*v(l) \\
& = i\left(\frac{\alpha}{2\pi}\right)\bar{u}(k)\int d^4p_\gamma d^4p_l\delta(P_1-p_\gamma-p_l)\delta(p_\gamma^2)\delta(p_l^2-m_l^2) \\
& \quad \otimes \sum_{Spin} [\bar{u}(k)\Gamma v(p_l)\cdot\bar{v}(p_l)\tilde{\epsilon}\left(\frac{l+q-m_l}{(l+q)^2-m_l^2}\right)\tilde{\epsilon}^*v(l)]
\end{aligned}$$

(b)



$$\begin{aligned}
M_b & = \bar{u}(k)\int \frac{d^4p_\gamma}{(2\pi)^4} \frac{d^4p_l}{(2\pi)^4} \left(\frac{i}{p_\gamma^2+i\epsilon}\right)\Gamma\left(\frac{i}{-p_l-m_l+i\epsilon}\right) \\
& \quad (-ie\tilde{\epsilon}^*)\left(\frac{i}{-p_l+q-m_l+i\epsilon}\right)(ie\tilde{\epsilon})v(l)(2\pi)^4\delta(P_1-p_\gamma-p_l) \\
& = -\frac{e^2}{2(2\pi)^2}\bar{u}(k)\int d^4p_\gamma d^4p_l\delta(P_1-p_\gamma-p_l)\delta(p_\gamma^2)\delta(p_l^2-m_l^2) \\
& \quad \otimes \sum_{Spin} [\bar{u}(k)\Gamma(p_l-m_l)\tilde{\epsilon}^*\left(\frac{i}{-p_l+q-m_l+i\epsilon}\right)\tilde{\epsilon}v(l)] \\
& = i\left(\frac{\alpha}{2\pi}\right)\int d^4p_\gamma d^4p_l\delta(P_1-p_\gamma-p_l)\delta(p_\gamma^2)\delta(p_l^2-m_l^2) \\
& \quad \otimes \sum_{Spin} [\bar{u}(k)\Gamma v(p_l)\cdot\bar{v}(p_l)\tilde{\epsilon}^*\left(\frac{p_l-q-m_l}{(p_l-q)^2-m_l^2}\right)\tilde{\epsilon}v(l)]
\end{aligned}$$

here

$$\begin{aligned}
\Gamma & = C_1(1+\gamma_5)\left(\frac{p\cdot\tilde{\epsilon}^*(p_\gamma)}{p\cdot p_\gamma} - \frac{p_\gamma\tilde{\epsilon}^*(p_\gamma)+2p_l\cdot\tilde{\epsilon}^*(p_\gamma)}{2p_l\cdot p_\gamma}\right) \\
& \quad + C_2\left\{(pp_\gamma)\frac{A}{M_K}(-\tilde{\epsilon}^*(p_\gamma)+\frac{p\cdot\tilde{\epsilon}^*(p_\gamma)p_\gamma}{(pp_\gamma)})+i\epsilon_{abcd}\frac{V}{M_K}p_\gamma^c p_\gamma^d \tilde{\epsilon}^{*a}(p_\gamma)\gamma^b\right\}(1-\gamma_5)
\end{aligned}$$

and

$$C_1 \equiv \frac{ieG_F}{\sqrt{2}} \sin \theta_c m_l f_K, \quad C_2 \equiv -\frac{ieG_F}{\sqrt{2}} \sin \theta_c$$

$$\text{Therefore } M^+ M_a = -\left(\frac{\alpha}{2\pi}\right) \otimes$$

$$\begin{aligned} & [C_1^* C_1 \bar{v}(l) \left( \frac{p\varepsilon}{pq} - \frac{\varepsilon q + 2l\varepsilon}{2(lq)} \right) (1 - \gamma_5) u(k) \bar{u}(k) (1 + \gamma_5) \otimes \\ & \{-C \cdot (p - k - m_l) p + (A \cdot M_K^2 + B k p) + \frac{1}{2(lq)} [(8H + 2IP_1^2 - 8F - 2GP_1^2) - (4m_l D + 2m_l E) P_1]\} \\ & + C_1^* C_2 \bar{v}(l) \left( \frac{p\varepsilon}{pq} - \frac{\varepsilon q + 2l\varepsilon}{2(lq)} \right) (1 - \gamma_5) u(k) \bar{u}(k) (1 + \gamma_5) \otimes \{ \\ & \quad \frac{A}{M_K} [-(6F + GP_1^2) p - 2G(pP_1) P_1 - 4m_l D(pP_1) + m_l D P_1 p] \\ & \quad - i\varepsilon_{abcd} \frac{V}{M_K} p^d \gamma^b (F\gamma^c + GP_1^c P_1 - m_l D P_1^c) \gamma^a \} \\ & + C_2^* C_1 \bar{v}(l) [(1 + \gamma_5) \left( (pq) \frac{A}{M_K} \left( -\varepsilon + \frac{(p\varepsilon)q}{pq} \right) - i\varepsilon_{\mu\nu\alpha\beta} \frac{V}{M_K} q^\alpha p^\beta \varepsilon^\mu \gamma^\nu \right) ] u(k) \bar{u}(k) (1 + \gamma_5) \otimes \\ & \{-C \cdot (p - k - m_l) p + (A \cdot M_K^2 + B k p) + \frac{1}{2(lq)} [(8H + 2IP_1^2 - 8F - 2GP_1^2) - (4m_l D + 2m_l E) P_1]\} \\ & + C_2^* C_2 \bar{v}(l) [(1 + \gamma_5) \left( (pq) \frac{A}{M_K} \left( -\varepsilon + \frac{(p\varepsilon)q}{pq} \right) - i\varepsilon_{\mu\nu\alpha\beta} \frac{V}{M_K} q^\alpha p^\beta \varepsilon^\mu \gamma^\nu \right) ] u(k) \bar{u}(k) (1 + \gamma_5) \\ & \otimes \left\{ \frac{A}{M_K} [-(6F + GP_1^2) p - 2G(pP_1) P_1 - 4m_l D(pP_1) + m_l D P_1 p] \right. \\ & \quad \left. - i\varepsilon_{abcd} \frac{V}{M_K} p^d \gamma^b (F\gamma^c + GP_1^c P_1 - m_l D P_1^c) \gamma^a \right\} \otimes \frac{(l + q - m_l)}{(l + q)^2 - m_l^2} \varepsilon^* v(l) \end{aligned}$$

$$\text{and } M^+ M_b = \left(\frac{\alpha}{2\pi}\right) \int d^4 p_\gamma d^4 p_l \delta(P_1 - p_\gamma - p_l) \delta(p_\gamma^2) \delta(p_l^2 - m_l^2) \otimes$$

$$\begin{aligned} & \{ C_1^* C_1 \bar{v}(l) \left( \frac{p\varepsilon}{pq} - \frac{\varepsilon q + 2l\varepsilon}{2(lq)} \right) (1 - \gamma_5) u(k) \bar{u}(k) (1 + \gamma_5) \otimes \\ & \quad \left( \frac{p \cdot \tilde{\varepsilon}^*(p_\gamma)}{p \cdot p_\gamma} - \frac{p_\gamma \cdot \tilde{\varepsilon}^*(p_\gamma) + 2p_l \cdot \tilde{\varepsilon}^*(p_\gamma)}{2p_l \cdot p_\gamma} \right) (p_l - m_l) \end{aligned}$$

$$\begin{aligned}
& +C_1^*C_2\bar{v}(l)(\frac{p\varepsilon}{pq}-\frac{\varepsilon q+2l\varepsilon}{2(lq)})(1-\gamma_5)u(k)\bar{u}(k)(1+\gamma_5)\otimes \\
& \quad \{(pp_\gamma)\frac{A}{M_K}(-\tilde{\varepsilon}^*(p_\gamma)+\frac{p\cdot\tilde{\varepsilon}^*(p_\gamma)p_\gamma}{(pp_\gamma)})+i\varepsilon_{abcd}\frac{V}{M_K}p_\gamma^cp_\gamma^d\tilde{\varepsilon}^{*a}(p_\gamma)\gamma^b\}(p_l-m_l) \\
& +C_2^*C_1\bar{v}(l)[(1+\gamma_5)\{(pq)\frac{A}{M_K}(-\varepsilon+\frac{(p\varepsilon)q}{pq})-i\varepsilon_{\mu\nu\alpha\beta}\frac{V}{M_K}q^\alpha p^\beta\varepsilon^\mu\gamma^\nu\}]\otimes \\
& \quad u(k)\bar{u}(k)(1+\gamma_5)(\frac{p\cdot\tilde{\varepsilon}^*(p_\gamma)}{p\cdot p_\gamma}-\frac{p_\gamma\tilde{\varepsilon}^*(p_\gamma)+2p_l\cdot\tilde{\varepsilon}^*(p_\gamma)}{2p_l\cdot p_\gamma})(p_l-m_l) \\
& +C_2^*C_2\bar{v}(l)[(1+\gamma_5)\{(pq)\frac{A}{M_K}(-\varepsilon+\frac{(p\varepsilon)q}{pq})-i\varepsilon_{\mu\nu\alpha\beta}\frac{V}{M_K}q^\alpha p^\beta\varepsilon^\mu\gamma^\nu\}]\otimes \\
& \quad u(k)\bar{u}(k)(1+\gamma_5)\{(pp_\gamma)\frac{A}{M_K}(-\tilde{\varepsilon}^*(p_\gamma)+\frac{p\cdot\tilde{\varepsilon}^*(p_\gamma)p_\gamma}{(pp_\gamma)})+i\varepsilon_{abcd}\frac{V}{M_K}p_\gamma^cp_\gamma^d\tilde{\varepsilon}^{*a}(p_\gamma)\gamma^b\}(p_l-m_l) \} \\
& \quad \otimes \varepsilon^*(\frac{p_l-q-m_l}{(p_l-q)^2-m_l^2})\tilde{\varepsilon}v(l)
\end{aligned}$$

We put  $u(k)\cdot\bar{u}(k)\rightarrow k$ ,  $v(l)\cdot\bar{v}(l)\rightarrow\frac{1}{2}(l-m_l)(1+\gamma_5\not{s})$  and  $v(p_l)\cdot\bar{v}(p_l)\rightarrow\frac{1}{2}(p_l-m_l)(1+\gamma_5\not{s})$

to the above equations and take trace to obtain the differential cross-section. We have calculated the imaginary part, i.e., the coefficient of  $\varepsilon^{\mu\nu\alpha\beta}p^\mu s^\nu q^\alpha l^\beta$ . There are four parts for each diagram. We will denote them as (a1), (a2), (a3) and (a4) for the first diagram and (b1), (b2), (b3) and (b4) for the second diagram. We list the phase space integrations involved and (a1), (a2),...(b4) in the Appendix.

## CHAPTER 6. DISCUSSIONS AND CONCLUSIONS

The transverse muon polarization  $P_{\perp}$  induced by the final state electromagnetic interaction is on the order of  $10^{-3}$  instead of  $10^{-4}$  [17]. This order of magnitude occurred over 50% of the phase space. We check that at some portions of the phase space the polarization becomes  $10^{-2}$ . This happens near the upper right hand corner of the Dalitz plot. That region corresponds to the case where both photon and lepton are energetic. But this behavior can be understood from the following argument. The photons we are dealing with are Bremsstrahlung photons (not primary photons), and the inner Bremsstrahlung photon is dominant in this process from the graph of the density function in Fig. 6.4. The number of events of high energy Bremsstrahlung photons is very small. That event rate is at least two orders of magnitude smaller than that of the low energy photons. Therefore, the polarization is expected to be roughly two orders higher than that in the low energy region. Although the polarization is large for the high energy photon region, the actual detection of the photon would be very difficult because the angle the photon makes to the emitter becomes almost co-linear (the angle is inversely proportional to the photon energy).

We include the Dalitz plot of the tree density function  $\rho(x,y)$  (Fig. 6.4) and the muon polarization  $P_{\perp}$  (Fig. 6.5)

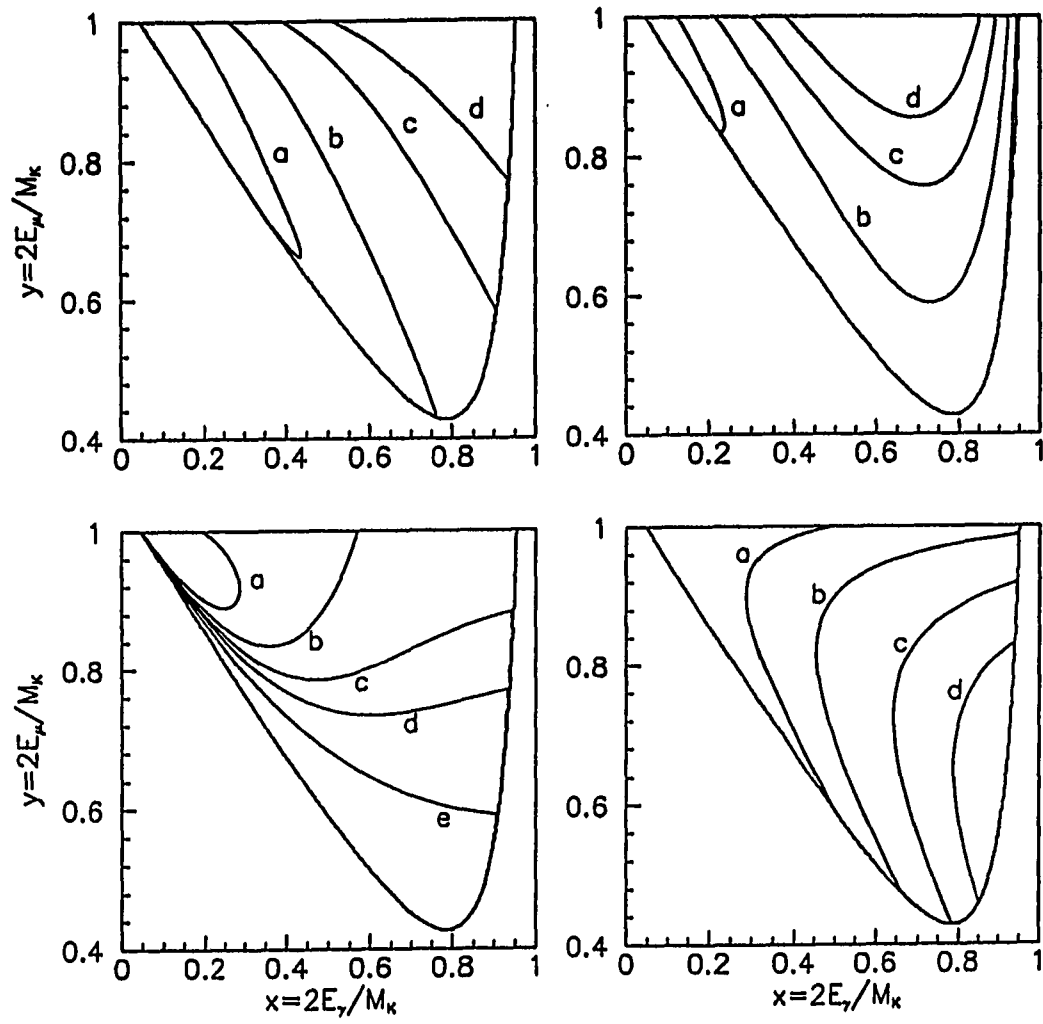
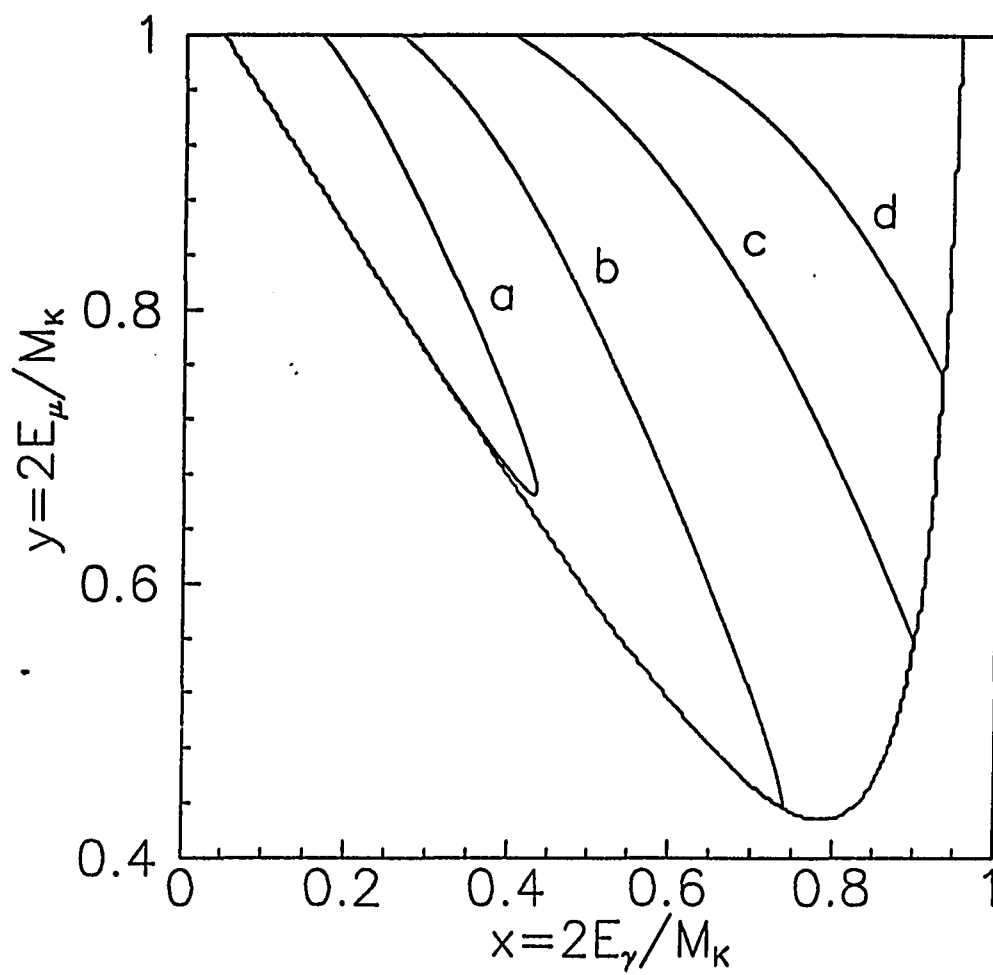


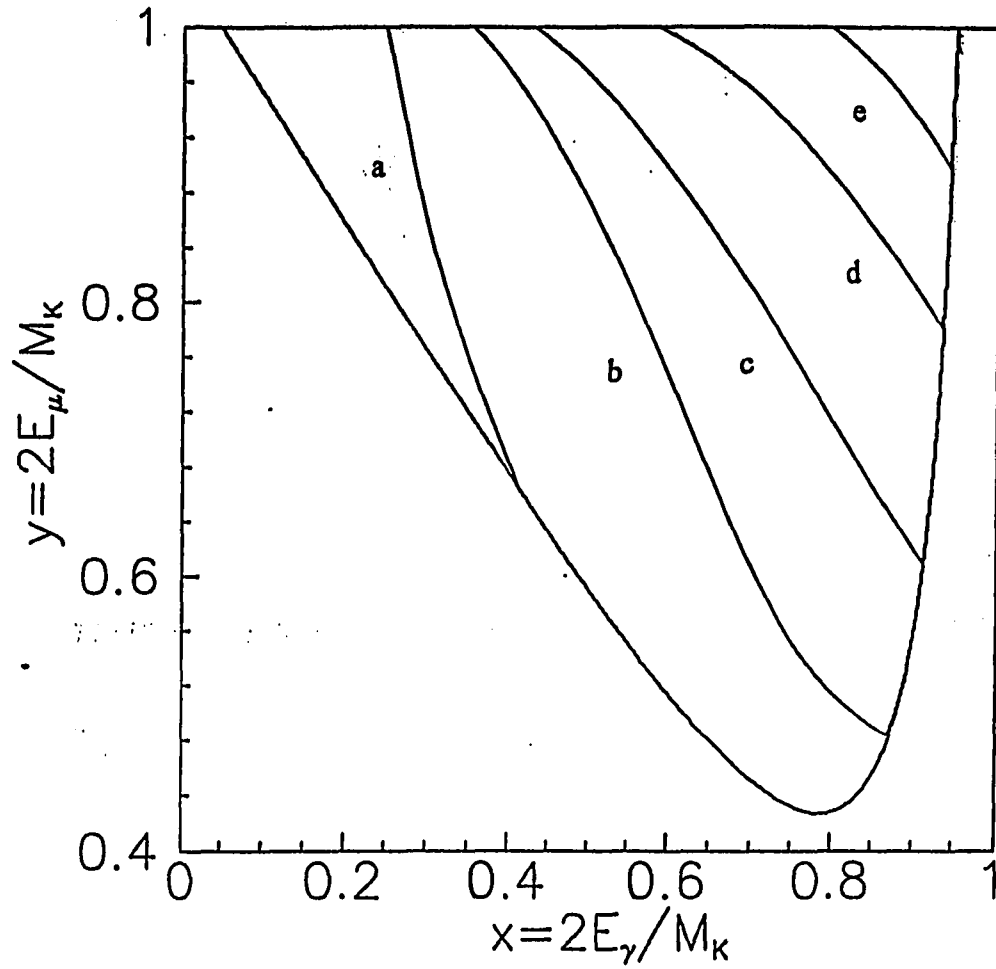
Fig. 6.4.a Dalitz plot of the density function  $\rho(x,y)$  of  $K_{l2\gamma}$





$$a = 0.2, b = 0.04, c = 0.01, d = 0.004$$

Fig. 6.4.b Dalitz plot of the dominant function  $\rho_{IB}(x,y)$



$$a = 0.0003, b = 0.0004, c = 0.001, d = 0.003, e = 0.008$$

Fig. 6.5 Dalitz plot of the muon polarization  $P_1$

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## APPENDIX

First Diagram:

$$\iint \frac{d\vec{p}_\gamma}{2E_\gamma} \frac{d\vec{p}_l}{2E_l} \frac{1}{p \cdot p_\gamma} \delta^{(4)}(p_l + p_\gamma - P_1) = \left(\frac{1}{2}\right)^2 \cdot 2\pi \cdot \frac{1}{pk} \cdot \ln\left(\frac{(pP_1)(1 - \frac{m_l^2}{P_1^2}) + (pk) \cdot \lambda_0}{(pP_1)(1 - \frac{m_l^2}{P_1^2}) - (pk) \cdot \lambda_0}\right)$$

$$\iint \frac{d\vec{p}_\gamma}{2E_\gamma} \frac{d\vec{p}_l}{2E_l} \frac{p_\mu}{p \cdot p_\gamma} \delta^{(4)}(p_l + p_\gamma - P_1) = A p_a + B k_a$$

$$\text{here } A = \frac{\pi}{4} \frac{1}{pk} \left\{ \frac{(pk) + m_l^2 - \frac{m_l^2(M_K^2 - pk)}{(M_K^2 - 2pk)}}{pk} \ln\left(\frac{(M_K^2 - pk) - \frac{m_l^2(M_K^2 - pk)}{(M_K^2 - 2pk)} + (pk) \cdot \lambda_0}{(M_K^2 - pk) - \frac{m_l^2(M_K^2 - pk)}{(M_K^2 - 2pk)} - (pk) \cdot \lambda_0}\right) + \right. \\ \left. \frac{1}{pk} (2\lambda_0 \cdot pk - (M_K^2 - pk - \frac{m_l^2(M_K^2 - pk)}{(M_K^2 - 2pk)})) \ln\left(\frac{(M_K^2 - pk) - \frac{m_l^2(M_K^2 - pk)}{(M_K^2 - 2pk)} + (pk) \cdot \lambda_0}{(M_K^2 - pk) - \frac{m_l^2(M_K^2 - pk)}{(M_K^2 - 2pk)} - (pk) \cdot \lambda_0}\right) \right\}$$

$$B = 4\pi \left(\frac{1}{2}\right)^3 \cdot \lambda_0 - M_K^2 \cdot A \quad \text{where } \lambda_0 \equiv \sqrt{1 + \left(\frac{m_l^2}{P_1^2}\right)^2 - \frac{2m_l^2}{P_1^2}}$$

$$W(a, b, c) = [a^2 + b^2 + c^2 - 2ac - 2ab - 2bc]^{1/2}$$

$$\iint \frac{d\vec{p}_\gamma}{2E_\gamma} \frac{d\vec{p}_l}{2E_l} p_{\mu} \delta^{(4)}(p_l + p_\gamma - P_1) = D P_{1a} \quad \text{where } D = \frac{\pi}{4} \frac{1}{P_1^4} W(P_1^2, m_l^2, 0) \cdot (P_1^2 - m_l^2)$$

$$\iint \frac{d\vec{p}_\gamma}{2E_\gamma} \frac{d\vec{p}_l}{2E_l} p_{\mu} \delta^{(4)}(p_l + p_\gamma - P_1) = E P_{1a} \quad \text{where } E = \frac{\pi}{4} \frac{1}{P_1^4} W(P_1^2, m_l^2, 0) \cdot (P_1^2 + m_l^2)$$

$$\iint \frac{d\vec{p}_\gamma}{2E_\gamma} \frac{d\vec{p}_l}{2E_l} p_{\mu} p_{\nu} \delta^{(4)}(p_l + p_\gamma - P_1) = F g^{ab} + G P_{1a} P_{1b}$$

$$\text{where } F = \frac{\pi}{24} \frac{1}{P_1^4} W^3(P_1^2, m_l^2, 0) \quad \text{and } G = \frac{\pi}{12} \frac{1}{P_1^6} W(P_1^2, m_l^2, 0) \cdot [P_1^4 + P_1^2 m_l^2 - 2m_l^4]$$

$$\iint \frac{d\vec{p}_\gamma}{2E_\gamma} \frac{d\vec{p}_l}{2E_l} p_{\mu} p_{\nu} \delta^{(4)}(p_l + p_\gamma - P_1) = H g^{ab} + I P_{1a} P_{1b}$$

$$\text{where } H = -\frac{\pi}{24} \frac{1}{P_1^6} W^3(P_1^2, m_l^2, 0) \quad \text{and } I = \frac{\pi}{6} \frac{1}{P_1^6} W(P_1^2, m_l^2, 0) \cdot [(P_1^2 + m_l^2)^2 - P_1^2 m_l^2]$$

$$\tilde{I} \equiv -C \cdot M_K^2 + A \cdot M_K^2 + \frac{1}{2lq} (8H - 8F + 2I \cdot P_1^2 - 2G \cdot P_1^2)$$

$$\tilde{II} \equiv m_l \cdot C - \frac{1}{lq} (2m_l \cdot D + m_l E)$$

$$\tilde{E} \equiv -(6F + G \cdot P_1^2) - 2G \cdot (pP_1)$$

$$\tilde{F} \equiv m_l \cdot D \cdot (M_K^2 - 4(pP_1))$$

$$(a1) = \frac{\alpha}{2\pi} |C_1|^2 \frac{2}{(lq \cdot pq)} (m_l \cdot \tilde{I} - \tilde{II} \cdot (M_K^2 - 2pk))$$

$$(a2) = \frac{\alpha}{2\pi} C_1^* \cdot C_2 \frac{1}{2lq} \left\{ \frac{A}{M_K} \left[ \frac{4}{pq} (m_l \cdot \tilde{F} - 2\tilde{E} \cdot (pl + pq - 5 \cdot M_K^2)) \right] \right.$$

$$\left. - \frac{V}{M_K} \left[ \frac{1}{pq} \{ F \cdot (-48(pl + pq) + 24M_K^2) + G \cdot (16(pl \cdot P_1 k - pq \cdot pk) - 8M_K^2(qk + kl)) \right. \right.$$

$$\left. - 8m_l^2 \cdot D \cdot (pk) \} \right] \}$$

$$(a3) = \frac{\alpha}{2\pi} C_2^* \cdot C_1 \frac{4}{lq} \left\{ \frac{A}{M_K} [(pl + qk) \cdot \tilde{I} - m_l \cdot (pl + pq) \cdot \tilde{II}] \right.$$

$$\left. + \frac{V}{M_K} [-(kl + qk) \cdot \tilde{I} - m_l \cdot (pk) \cdot \tilde{II}] \right\}$$

$$(a4) = \frac{\alpha}{2\pi} |C_2|^2 \frac{1}{2lq} \left\{ \left( \frac{A}{M_K} \right)^2 [-8m_l \tilde{E} \cdot (pl + pq) + \tilde{F} \cdot (pl + qk)] \right.$$

$$\left. - \frac{A \cdot V}{M_K^2} [-48 \cdot m_l \cdot F \cdot (pl + pq) + 16 \cdot m_l \cdot G \cdot pk \cdot (pl + pq) \right.$$

$$\left. + 16 \cdot m_l \cdot D \cdot (-(pl + pq)(qk + lk) + qk \cdot pk) \right]$$

$$\left. - \frac{A \cdot V}{M_K^2} [-8 \cdot m_l \cdot \tilde{E} \cdot pk + 8 \cdot \tilde{F} \cdot (lk + pq - lq)] \right.$$

$$\left. - m_l \cdot \left( \frac{V}{M_K} \right)^2 [48 \cdot F \cdot pk - 16G \cdot (pk)^2 + 8D \cdot (-pq \cdot lq - 2m_l^2 \cdot pl + pl \cdot pq - 2qk \cdot pk)] \right\}$$

Second Diagram:

$$\lambda_0 = 1 - \frac{m_l^2}{P_1^2}$$

$$\Delta \equiv \iint d^4 p_l d^4 p_\gamma \delta(p_\gamma^2) \delta(p_l^2 - m_l^2) \delta^{(4)}(P_1 - p_\gamma - p_l) = \frac{\pi}{2} \lambda_0$$

$$CA \equiv \iint d^4 p_l d^4 p_\gamma \frac{1}{(pp_\gamma)(-2p_l q)} \delta(p_\gamma^2) \delta(p_l^2 - m_l^2) \delta^{(4)}(P_1 - p_\gamma - p_l)$$

$$\begin{aligned}
&= -2\pi\left(\frac{1}{2}\right)^2 \frac{1}{pk} \frac{1}{\sqrt{c}} \left\{ \sinh^{-1}\left(\frac{2c\frac{1}{\eta-1}+b}{\sqrt{\Pi}}\right) - \sinh^{-1}\left(\frac{2c\frac{1}{\eta+1}+b}{\sqrt{\Pi}}\right) \right\} \\
\text{CB} &\equiv \iint d^4 p_l d^4 p_\gamma \frac{1}{(-2p_l q)} \delta(p_\gamma^2) \delta(p_l^2 - m_l^2) \delta^{(4)}(P_1 - p_\gamma - p_l) \\
&= 2\pi\left(\frac{1}{2}\right)^3 \frac{1}{lq} \ln\left(\frac{1 + \frac{m_l^2}{P_1^2} - \lambda_0}{1 + \frac{m_l^2}{P_1^2} + \lambda_0}\right)
\end{aligned}$$

$$K \cdot \Lambda = x(Kq) - (1-x)(Kp)$$

$$K \cdot \Sigma = x(Kq) + (1-x)(Kp)$$

$$p \cdot \Lambda = x(pq) - (1-x)M_K^2$$

$$\Lambda^2 = M_K^2(1-x)^2 - 2x(1-x)(pq)$$

$$\Sigma^2 = M_K^2(1-x)^2 + 2x(1-x)(pq)$$

$$O = \sqrt{(K \cdot \Lambda)^2 - K^2 \Lambda^2}$$

$$D_- = K \cdot \Sigma + K \cdot \Lambda \left(\frac{m_l^2}{K^2}\right) - \lambda_0 \cdot O$$

$$D_+ = K \cdot \Sigma + K \cdot \Lambda \left(\frac{m_l^2}{K^2}\right) + \lambda_0 \cdot O$$

$$N_- = (Kp)\left(1 + \frac{m_l^2}{K^2}\right) - \lambda_0((K \cdot \Lambda)(Kp) - K^2(p \cdot \Lambda))/O$$

$$N_+ = (Kp)\left(1 + \frac{m_l^2}{K^2}\right) + \lambda_0((K \cdot \Lambda)(Kp) - K^2(p \cdot \Lambda))/O$$

$$\text{CC} \equiv \iint d^4 p_l d^4 p_\gamma \frac{pp_l}{(pp_\gamma)(-2p_l q)} \delta(p_\gamma^2) \delta(p_l^2 - m_l^2) \delta^{(4)}(P_1 - p_\gamma - p_l)$$

$$= -\frac{\pi}{4} \lambda_0 \int_0^1 \left\{ \frac{1}{\lambda_0 O} \left( \frac{N_-}{D_-} - \frac{N_+}{D_+} \right) - \frac{1}{\lambda_0 O^3} ((K \cdot \Lambda)(Kp) - K^2(p \cdot \Lambda)) \cdot \ln\left(\frac{D_-}{D_+}\right) \right\}$$

$$\tilde{A} = -\frac{\pi}{4} \lambda_0 \left\{ \frac{-(1-x)K^2}{O^2} \left( \frac{1}{D_-} + \frac{1}{D_+} \right) - \frac{-(1-x)K^2}{\lambda_0 O^3} \cdot \ln\left(\frac{D_-}{D_+}\right) \right\}$$

$$\tilde{B} = -\frac{\pi}{4} \lambda_0 \left\{ \frac{1}{O} \left[ \frac{(1 + \frac{m_l^2}{K^2} - \lambda_0 \frac{K \cdot \Lambda}{O})}{D_-} - \frac{(1 + \frac{m_l^2}{K^2} + \lambda_0 \frac{K \cdot \Lambda}{O})}{D_+} \right] - \frac{K \cdot \Lambda}{O^3} \ln\left(\frac{D_-}{D_+}\right) \right\}$$

$$\tilde{C} = -\frac{\pi}{4} \left\{ \frac{xK^2 \lambda_0}{O^2} \left( \frac{1}{D_-} + \frac{1}{D_+} \right) + \frac{xK^2}{O^3} \cdot \ln\left(\frac{D_-}{D_+}\right) \right\}$$

$$A = (Kp) \cdot \tilde{A}$$

$$B = (Kp) \cdot \tilde{B} + \frac{\pi}{4(Kq)} \lambda_0$$

$$C = (Kp) \cdot \tilde{C} + \frac{\pi}{8(Kq)^2} \lambda_0 K^2 \left( \frac{-(1 + \frac{m_l^2}{K^2})}{\lambda_0} \cdot \ln(\frac{m_l^2}{K^2}) - 2 \right)$$

$$A_3 = \frac{-\pi}{32(Kq)} K^2 \lambda_0 \left\{ \left( 1 - \frac{(1 + \frac{m_l^2}{K^2})^2}{\lambda_0^2} \right) \cdot \ln(\frac{m_l^2}{K^2}) - 2 \frac{(1 + \frac{m_l^2}{K^2})}{\lambda_0} \right\}$$

$$B_3 = -\frac{\pi}{32} \left\{ -[2(Kp)(1 + \frac{m_l^2}{K^2})^2 + \lambda_0^2 ((Kp) - \frac{((Kp)(Kq) - K^2(pq))}{(Kq)})] \frac{1}{(Kq)} \ln(\frac{m_l^2}{K^2}) \right.$$

$$+ 2\lambda_0 [(Kp) + \frac{((Kp)(Kq) - K^2(pq))}{(Kq)}] (1 + \frac{m_l^2}{K^2}) (\frac{2}{(Kq)} + \frac{(1 + \frac{m_l^2}{K^2})}{(Kq)\lambda_0} \ln(\frac{m_l^2}{K^2}))$$

$$- \lambda_0^2 (3 \frac{((Kp)(Kq) - K^2(pq))}{(Kq)} - (Kp)) (\frac{(1 + \frac{m_l^2}{K^2})^2}{(Kq)\lambda_0^2} \ln(\frac{m_l^2}{K^2}) + 2 \frac{(1 + \frac{m_l^2}{K^2})}{(Kq)\lambda_0}) \}$$

$$C_3 = -\frac{\pi}{32} \left\{ -K^2 \lambda_0^2 \frac{((Kp)(Kq) - K^2(pq))}{(Kq)^3} \ln(\frac{m_l^2}{K^2}) \right.$$

$$- 2(Kp)K^2 (1 + \frac{m_l^2}{K^2}) \frac{\lambda_0}{(Kq)} (\frac{2}{(Kq)} + \frac{(1 + \frac{m_l^2}{K^2})}{(Kq)\lambda_0} \cdot \ln(\frac{m_l^2}{K^2}))$$

$$+ 3K^2 \lambda_0^2 \frac{((Kp)(Kq) - K^2(pq))}{(Kq)^2} (\frac{(1 + \frac{m_l^2}{K^2})^2}{(Kq)\lambda_0^2} \cdot \ln(\frac{m_l^2}{K^2}) + \frac{(1 + \frac{m_l^2}{K^2})}{(Kq)\lambda_0}) \}$$

$$da40 = (1 + \frac{m_l^2}{K^2})$$

$$da4 = (Kq)(1 + \frac{m_l^2}{K^2})$$

$$db4 = -(Kq)\lambda_0$$

$$dc4 = \frac{((Kp)(Kq) - K^2(pq))}{(Kq)}$$

$$\alpha 4 = \{ (Kp)^2 (da40)^3 + (3/2) \lambda_0^2 (da40) (Kp)^2 - (Kp) (da40) \lambda_0^2 (dc4) -$$

$$.5 \lambda_0^2 (da40) (M_K^2 K^2 + (dc4)^2) \}$$

$$\alpha t4 = [(Kp) (da40) \lambda_0^2 (dc4) K^2] / (Kq)$$



$$\beta_4 = \{-3(Kp)^2(da40)^2\lambda_0 + 2[(Kp)(da40)\lambda_0(pq)K^2]/(Kq) - \lambda_0^3((Kp)^2 - K^2 M_K^2 - 2(dc4)^2) - \lambda_0^3(Kp)(dc4)\}$$

$$\beta_{t4} = \{(Kp)^2(da40)^2\lambda_0 K^2/(Kq) + \lambda_0^3((Kp)^2 - K^2 M_K^2 - 2(dc4)^2)K^2/(Kq)\}$$

$$\gamma_4 = \{-(3/2)(Kp)^2(da40)\lambda_0^2 + 3(Kp)(da40)(dc4)\lambda_0^2 + .5(da40)\lambda_0^2(K^2 M_K^2 + 3(dc4)^2)\}$$

$$\gamma_{t4} = -3\lambda_0^2(Kp)(da40)(dc4)K^2/(Kq)$$

$$\delta_4 = \lambda_0^3\{.5((Kp)^2 - K^2 M_K^2) + (Kp)(dc4) - 2.5(dc4)^2\}$$

$$\delta_{t4} = -.5((Kp)^2 - K^2 M_K^2) K^2/(Kq) + 2.5\lambda_0^3(dc4)K^2/(Kq)$$

$$A_4 = (\pi/32)\lambda_0(1/db4)\{(1/db4)[(da4+db4)\ln(da4+db4)-(da4-db4)\ln(da4-db4)-2db4]\lambda_0^3(dc4)K^2 + [(1-(da4/db4)^2)\ln((da4+db4)/(da4-db4)) + 2(da4/db4)](Kp)(1+\frac{m_l^2}{K^2})K^2\lambda_0^2 +$$

$$-[(1+(da4/db4)^3)\ln(da4+db4) - (-1+(da4/db4)^3)\ln(da4-db4) - (2/3) - 2(da4/db4)^2]\lambda_0^3 K^2(dc4)\}$$

$$B_4 = -(\pi/32)\lambda_0(1/db4)\{[\ln(da4+db4)(\alpha_4+\beta_4+\gamma_4+\delta_4) - \ln(da4-db4)(\alpha_4-\beta_4+\gamma_4-\delta_4)]$$

$$-[(1/db4)((da4+db4)\ln(da4+db4)-(da4-db4)\ln(da4-db4) - 2db4)\beta_4$$

$$+((1-(da4/db4)^2)\ln((da4+db4)/(da4-db4)) + 2(da4/db4))\gamma_4$$

$$+((1+(da4/db4)^3)\ln(da4+db4) - (-1+(da4/db4)^3)\ln(da4-db4) - (2/3) - 2(da4/db4)^2)\delta_4\}$$

$$C_4 = (\pi/32)\lambda_0(1/db4)\{\ln(da4+db4)(\alpha_{t4}+\beta_{t4}+\gamma_{t4}+\delta_{t4}) - \ln(da4-db4)(\alpha_{t4}-\beta_{t4}+\gamma_{t4}-\delta_{t4})$$

$$-[(1/db4)((da4+db4)\ln(da4+db4)-(da4-db4)\ln(da4-db4) - 2db4)\beta_{t4}$$

$$+((1-(da4/db4)^2)\ln((da4+db4)/(da4-db4)) + 2(da4/db4))\gamma_{t4}$$

$$+((1+(da4/db4)^3)\ln(da4+db4) - (-1+(da4/db4)^3)\ln(da4-db4) - (2/3) - 2(da4/db4)^2)\delta_{t4}\}$$

$$CD \equiv \iint d^4 p_l d^4 p_\gamma \frac{1}{(pp_\gamma)} \delta(p_\gamma^2) \delta(p_l^2 - m_l^2) \delta^{(4)}(P_1 - p_\gamma - p_l)$$

$$= 2\pi\left(\frac{1}{2}\right)^2 \frac{1}{pk} \ln\left(\frac{P_1 p \cdot (1 - \frac{m_l^2}{P_1^2}) + pk \cdot \lambda_0}{P_1 p \cdot (1 - \frac{m_l^2}{P_1^2}) - pk \cdot \lambda_0}\right)$$

where

$$a = (P_1 q)^2 \cdot \lambda_0^2$$

$$b = -2\{(P_1 q)^2 \cdot \lambda_0^2 \cdot \eta + \lambda_0 \frac{(P_1 q \cdot P_1 p - P_1^2 \cdot pq)}{pk} (P_1 q)(1 + \frac{m_l^2}{P_1^2})\}$$

$$c = (P_1 q)^2 \cdot \lambda_0^2 \cdot \eta^2 + 2\lambda_0 \frac{(P_1 q \cdot P_1 p - P_1^2 \cdot pq)}{pk} (P_1 q)(1 + \frac{m_l^2}{P_1^2}) \cdot \eta + (P_1 q)^2 (1 + \frac{m_l^2}{P_1^2})^2 \\ - \lambda_0^2 \cdot \frac{(P_1 q)^2 \cdot (pk)^2 - (P_1 q \cdot P_1 p - P_1^2 \cdot pq)^2}{(pk)^2}$$

$$Q = (P_1 p \cdot (1 + \frac{m_l^2}{P_1^2}) + pk \cdot \lambda_0 \cdot \eta)$$

$$\Pi = 4ac - b^2$$

$$\eta = \frac{P_1 p \cdot (1 - \frac{m_l^2}{P_1^2})}{pk \cdot \lambda_0}$$

$$\iint d^4 p_l d^4 p_\gamma \frac{P_{la}}{(-2p_l q)} \delta(p_\gamma^2) \delta(p_l^2 - m_l^2) \delta^{(4)}(P_1 - p_\gamma - p_l) \equiv \alpha \cdot P_{la} + \beta \cdot q_a$$

$$\text{where } \alpha = -2\pi\left(\frac{1}{2}\right)^4 \left(\frac{2}{P_1 q}\right) \text{ and } \beta = 2\pi\left(\frac{1}{2}\right)^4 \lambda_0^2 \left(\frac{P_1^2}{P_1 q^2}\right) \left\{ \frac{2}{\lambda_0} + \frac{(1 + \frac{m_l^2}{P_1^2})}{\lambda_0^2} \ln\left(\frac{(1 + \frac{m_l^2}{P_1^2}) - \lambda_0}{(1 + \frac{m_l^2}{P_1^2}) + \lambda_0}\right) \right\}$$

$$F \equiv \iint d^4 p_l d^4 p_\gamma \frac{pp_l}{(-2p_l q)} \delta(p_\gamma^2) \delta(p_l^2 - m_l^2) \delta^{(4)}(P_1 - p_\gamma - p_l)$$

$$= -\frac{\pi}{8} \frac{\lambda_0}{lq} \{P_1 p \cdot (1 + \frac{m_l^2}{P_1^2}) \frac{1}{\sqrt{c_1}} [\sinh^{-1}(\frac{2c_1 + b_1}{\sqrt{\Pi_1}}) - \sinh^{-1}(\frac{-2c_1 + b_1}{\sqrt{\Pi_1}})] \\ - pk \cdot \lambda_0 \cdot [\frac{1}{c_1} (\sqrt{c_1 + b_1 + a_1} - \sqrt{c_1 - b_1 + a_1}) - \frac{b_1}{2c_1} \frac{1}{\sqrt{c_1}} [\sinh^{-1}(\frac{2c_1 + b_1}{\sqrt{\Pi_1}}) - \sinh^{-1}(\frac{-2c_1 + b_1}{\sqrt{\Pi_1}})]]\}$$

$$a_1 = (P_1 q)^2 (1 + \frac{m_l^2}{P_1^2}) - \lambda_0^2 \frac{(P_1 q)^2 (pk)^2 - [P_1 q \cdot P_1 p - P_1^2 \cdot pq]^2}{(pk)^2}$$

$$b_1 = -2\lambda_0 (P_1 q) \frac{(P_1 q \cdot P_1 p - P_1^2 \cdot pq)}{(pk)} (1 + \frac{m_l^2}{P_1^2})$$

$$c_1 = (P_1 q)^2 \lambda_0^2$$

$$\Pi_1 = 4a_1 c_1 - b_1^2$$

$$J \equiv \iint d^4 p_l d^4 p_\gamma p p_l \delta(p_\gamma^2) \delta(p_l^2 - m_l^2) \delta^{(4)}(P_1 - p_\gamma - p_l) = \frac{\pi}{4} \lambda_0 \cdot P_1 p \cdot (1 + \frac{m_l^2}{P_1^2})$$

$$N \equiv \iint d^4 p_l d^4 p_\gamma (p_l q) \delta(p_\gamma^2) \delta(p_l^2 - m_l^2) \delta^{(4)}(P_1 - p_\gamma - p_l) = \frac{\pi}{4} \lambda_0 \cdot P_1 q \cdot (1 + \frac{m_l^2}{P_1^2})$$

$$\iint d^4 p_l d^4 p_\gamma (p p_l \cdot p_{1a}) \delta(p_\gamma^2) \delta(p_l^2 - m_l^2) \delta^{(4)}(P_1 - p_\gamma - p_l) \equiv A_5 P_a + B_5 P_{1a}$$

$$\text{where } A_5 = -\frac{\pi}{24} \lambda_0^3 \cdot P_1^2 \text{ and } B_5 = \frac{\pi}{8} \lambda_0 \cdot P_1 p \cdot ((1 + \frac{m_l^2}{P_1^2}) + \frac{1}{3} \lambda_0^2)$$

$$\iint d^4 p_l d^4 p_\gamma (p_l q \cdot p_{1a}) \delta(p_\gamma^2) \delta(p_l^2 - m_l^2) \delta^{(4)}(P_1 - p_\gamma - p_l) \equiv A_6 P_{1a} + B_6 q_{1a}$$

$$\text{where } A_6 = \frac{\pi}{8} \lambda_0 \cdot P_1 p \cdot ((1 - (\frac{m_l^2}{P_1^2})^2) - \frac{1}{3} \lambda_0^2) \text{ and } B_6 = \frac{\pi}{24} \lambda_0^3 \cdot P_1^2$$

$$\iint d^4 p_l d^4 p_\gamma (p_{1a}) \delta(p_\gamma^2) \delta(p_l^2 - m_l^2) \delta^{(4)}(P_1 - p_\gamma - p_l) \equiv \tilde{G} P_{1a} \text{ where } \tilde{G} = \frac{P_1^2 - P_1 l}{P_1^2} \cdot \Delta$$

$$\iint d^4 p_l d^4 p_\gamma (p_{1a}) \delta(p_\gamma^2) \delta(p_l^2 - m_l^2) \delta^{(4)}(P_1 - p_\gamma - p_l) \equiv G P_{1a} \text{ where } G = \frac{P_1 l}{P_1^2} \cdot \Delta$$

$$(b1) = (\frac{\alpha}{2\pi}) |C_1|^2 \{ \frac{4m_l}{pq} [2(C - pq \cdot \tilde{C}) - M_K^2 \cdot (\tilde{A} + \tilde{B}) + 2 \cdot CC + (M_K^2 - 2 \cdot pq) \cdot CA]$$

$$+ \frac{4m_l}{(pq \cdot lq)} [(2m_l^2 + lq) \cdot CB + 2C_3 + (qk - pq) \cdot (\alpha + \beta) + .5 \cdot \Delta]$$

$$- \frac{4m_l}{(lq)} [2 \cdot P_1 l \cdot \tilde{C} + CC - pl \cdot \tilde{B} - pq(\tilde{A} + \tilde{B} + \tilde{C}) + .5 \cdot CD + (-2 \cdot lq + 2 \cdot pl - m_l^2) \cdot CA]$$

$$+ \frac{4m_l}{(lq)^2} [G + m_l^2 \cdot CB - 2lq \cdot (\alpha + \beta) + 2P_1 l \cdot \beta + 2 \cdot P_1 l \cdot CB + \Delta - m_l^2 \cdot \alpha]$$

$$- \frac{4m_l}{(lq \cdot pq)} [ -((P_1 \cdot (k + l)) \cdot \alpha - B_3 - (qk + P_1 q) \cdot \beta$$

$$+ 3F - 3A_3 - 3B_3 - 2C_3 - 2pq \cdot CB + 2pq \cdot (\alpha + \beta)]$$

$$+ \frac{4m_l}{(lq)^2} [2P_1 l \cdot \beta - A_3 - (P_1 q \cdot (\alpha + \beta) + .5 \cdot G) - (P_1 l \cdot (CB - \alpha) + .5 \cdot (\Delta - G))$$

$$+ 4(P_1 l \cdot (CB - \alpha - \beta) + .5 \cdot (\Delta - G)) - 2lq \cdot (CB - \alpha - \beta)]$$

$$- \frac{4m_l}{(lq)} [-2 \cdot (B - P_1 l \cdot \tilde{B}) - m_l^2 \cdot CA + \frac{m_l^2 \cdot \alpha}{lq} + CC - P_1 l \cdot CA - .5 \cdot CD + qk \cdot (\tilde{B} + \tilde{C})$$

$$\begin{aligned}
& -pl \cdot (A+B) - \frac{2A_3}{lq} + \frac{G}{lq} + \frac{m_l^2 \cdot CB}{lq} \cdot (\tilde{B} + \tilde{C}) + 2qk \cdot CA] \\
& + \frac{4m_l^2}{(lq)^2} [-\tilde{G} - 2A_3 + G + m_l^2(CB - \alpha)] \\
(b2) = & -\frac{\alpha}{2\pi} C_1^* C_2 \left\{ -\frac{A}{M_K} \left[ \frac{1}{pq} \{ 16(P_1 p \cdot (A_3 + B_3 + C_3)) - (A_4 + B_4 + C_4) \right. \right. \\
& \quad \left. \left. - 8m_l^2 \cdot (P_1 p \cdot CB - F) - 8qk \cdot (P_1 p \cdot (\alpha + \beta) - A_3 - B_3) \right. \right. \\
& \quad \left. \left. - 8(P_1 p \cdot P_1 l \cdot CB + .5 \cdot P_1 p \cdot \Delta - P_1 l \cdot F - .5 \cdot J) \right\} \right. \\
& \quad \left. - \frac{1}{lq} \{ 16(P_1 p \cdot P_1 l \cdot (\alpha + \beta) + .5 \cdot P_1 p \cdot G - P_1 l \cdot (A_3 + B_3 + C_3) - .5 \cdot (A_5 + B_5)) \right. \right. \\
& \quad \left. \left. - 16(P_1 p \cdot P_1 l \cdot CB + .5 \cdot P_1 p \cdot \Delta - P_1 l \cdot F - .5 \cdot J) + 8m_l^2(P_1 p \cdot \alpha - (A_3 + B_3)) \right\} \right. \\
& \quad \left. + 8 \frac{m_l^2}{pq} (P_1 p \cdot CB - F) \right] \\
& + \frac{1}{pq} \{ 8(-P_1 k \cdot (B_3 + C_3) + I - P_1 l \cdot F + lq \cdot F - pl \cdot C_3) - 4m_l^2 M_K^2 (CB - \alpha - \beta) \\
& \quad - 8pq \cdot (-P_1 k \cdot (\alpha + \beta) + F - P_1 l \cdot CB + lq \cdot CB - pl \cdot \beta) + 4M_K^2 \cdot lq \cdot (CB - \beta) \} \\
& - \frac{1}{lq} \{ 8(-P_1 k \cdot (P_1 l \cdot (\alpha + \beta) + .5 \cdot G) + P_1 l \cdot F - (P_1 l)^2 \cdot CB + .5 \cdot J - .5 \cdot P_1 l \cdot \Delta \\
& \quad + lq \cdot (P_1 l \cdot CB + .5 \cdot \Delta) - pl \cdot P_1 l \cdot \beta) - 8m_l^2 \cdot pl \cdot (CB - \alpha - \beta) + m_l^4 \cdot \beta \\
& \quad - 8lq \cdot (-P_1 k \cdot (\alpha + \beta) + F - P_1 l \cdot CB + lq \cdot CB - pl \cdot \beta) \\
& \quad - 4m_l^2 \cdot (-P_1 k \cdot \alpha + F - P_1 l \cdot CB + lq \cdot CB - pq \cdot \beta) \\
& \quad - \frac{4m_l^2}{lq} (P_1 k \cdot (\alpha + \beta) - F + P_1 l \cdot CB - lq \cdot CB - pq \cdot \beta) \\
& + \frac{4m_l^2}{pq} (-2C_3 + pq \cdot \beta - (P_1 k + P_1 l) \cdot (\alpha + \beta) + lk \cdot \beta - A_3 + P_1 k \cdot CB - lk \cdot (CB - \alpha) - P_1 l \cdot CB + .5 \cdot \Delta - pq \cdot (CB - \alpha - \beta)) \\
& - \frac{4m_l^2 M_K^2}{pq} \cdot \beta - 4 \frac{m_l^2}{lq} (2pl \cdot (CB - \alpha - \beta) - m_l^2 \cdot \beta) \} \\
& + \frac{V}{M_K} \left\{ \frac{1}{pq} [16(I - B_4 - C_4 - P_1 p \cdot (F - B_3 - C_3) - P_1 l \cdot C_3) - 8(ql + pq - qk) \cdot C_3 + 16m_l^2 \cdot C_3] \right. \\
& \quad \left. - \frac{1}{lq} [16(-(P_1 l)^2 \cdot \beta - .5 \cdot B_6 + lq \cdot P_1 l \cdot \beta) + 16(P_1 l \cdot (F - .5 \cdot \Delta) - (P_1 l)^2 \cdot (CB + \alpha + \beta) - .5 \cdot P_1 l \cdot G \right. \right. \\
& \quad \left. \left. + .5 \cdot (N - A_6) - lq \cdot (F - P_1 l \cdot CB - B - C + P_1 l \cdot (\alpha + \beta)) \right. \right. \\
& \quad \left. \left. - 8(-lq \cdot (F - P_1 l \cdot CB - B - C + P_1 l \cdot (\alpha + \beta)) + .5 \cdot lk \cdot \tilde{G} + qk(P_1 l \cdot (CB - \alpha - \beta) + .5 \cdot (\Delta - G))) \right] \right\}
\end{aligned}$$

$$\begin{aligned}
& +16m_l^2 \cdot \beta \} \\
& + \frac{1}{2lq} \frac{A}{M_K} \{ -16(P_1 p \cdot G - (A_5 + B_6) + P_1 p \cdot P_1 l \cdot \alpha - P_1 l \cdot (A_3 + B_3)) - 32qk \cdot (P_1 p \cdot \alpha - A_3 - B_3) \\
& + 8(-J - P_1 l \cdot \tilde{G} + (P_1 k + P_1 l) \cdot G) + 16m_l^2 ((P_1 k + P_1 l) \cdot CB - F) \\
& - 16 \cdot lq \cdot (qk \cdot (\alpha + \beta) + pl \cdot \alpha) \\
& - 16(-(P_1 q \cdot (B_3 + C_3) - P_1 q \cdot P_1 l \cdot (\alpha + \beta)) + 5 \cdot P_1 l \cdot G + 5 \cdot (P_1 l \cdot \tilde{G} - J) + pl \cdot A_3) \\
& + 8m_l^2 ((P_1 q - P_1 k - P_1 l) \cdot CB + F + 5 \cdot \Delta + (qk + pl) \cdot (CB - \alpha - \beta)) + 8(P_1 k \cdot \Delta + (qk + pl) \cdot \tilde{G}) \\
& + 16(-P_1 q \cdot (F - P_1 l \cdot CB) - qk \cdot (P_1 q \cdot (\alpha + \beta) + 5 \cdot G) + pl \cdot (P_1 q \cdot \alpha + 5 \cdot G)) \\
& + 16qk \cdot (lq \cdot CB - 5 \cdot \tilde{G} - (P_1 q \cdot (\alpha + \beta) + 5 \cdot G)) \\
& - 16m_l^2 ((P_1 k + P_1 l) \cdot \alpha - B_3) + 8m_l^2 (P_1 q \cdot \beta - lq \cdot CB) \} - \frac{8}{lq} \frac{V}{M_K} (P_1 l \cdot \tilde{G} + B_5 + m_l^2 \cdot ql \cdot \beta) \\
(b3) = & \frac{\alpha}{2\pi} C_2^* C_1 \{ -pq \cdot \frac{A}{M_K} \{ 16(-(B + C - P_1 l \cdot (\tilde{B} + \tilde{C})) + CB - P_1 l \cdot CA) - 8m_l^2 \cdot \tilde{C} + \frac{1}{lq} (-16A_3 + 8m_l^2 \cdot \beta) \} \\
& + \frac{A}{M_K} \{ 8(qk \cdot (-B - C + pq \cdot (\tilde{B} + \tilde{C})) + 5 \cdot ((P_1 p - pq) \cdot CD - \Delta) - CE + (P_1 l + pq) \cdot CC \\
& - P_1 l \cdot CA + pl \cdot (-A - B + pq \cdot (\tilde{A} + \tilde{B}))) + 8M_K^2 (qk \cdot \tilde{A} + 5 \cdot CD) + 8m_l^2 (pq \cdot CA - CC) \\
& - \frac{1}{lq} (-8m_l^2 \cdot F + 16qk \cdot A_3 - 4((qk - pq) \cdot G + N - P_1 l \cdot \Delta) \\
& - 8m_l^2 \cdot ((qk - pq) \cdot CB + A_3 + B_3 + (qk - pq) \cdot \alpha)) \} \\
& - \frac{V}{M_K} \{ -8(m_l^2 \cdot pl \cdot CA + 5 \cdot pk \cdot CD - qk \cdot (CC - P_1 l \cdot CA) + qk \cdot CC) \\
& - 8pl \cdot (B - P_1 l \cdot \tilde{B}) + 8pk \cdot (P_1 l \cdot \tilde{B} - E) - 8kl \cdot B \\
& - \frac{1}{lq} (4kl \cdot G - 8J + P_1 l \cdot \Delta - 8ql \cdot (B_3 - P_1 l \cdot \alpha) + 8qk \cdot (P_1 l \cdot \alpha + 5 \cdot G) - 8m_l^2 \cdot qk \cdot CB) \\
& + 8 \frac{m_l^2}{lq} (B_3 - P_1 l \cdot \alpha) - 8m_l^2 \cdot pk \cdot \tilde{B} + 8m_l^2 \frac{qk}{lq} \cdot \alpha + 8m_l^2 \cdot pk \cdot CA + 8m_l^2 \frac{lk}{lq} \cdot \alpha \} \\
& + \frac{pq}{2lq} \frac{A}{M_K} \{ -16(P_1 l \cdot (CB - \alpha) + 5 \cdot \Delta) - m_l^2 \cdot (32\alpha - 16(CB - \alpha)) \} \\
& - \frac{1}{2lq} \frac{A}{M_K} \{ 16(P_1 q \cdot (A_3 + B_3 + C_3) + 5 \cdot (A_5 + 2B_5) + (P_1 k + 2P_1 l) \cdot A_3 - A_4) \\
& + 16qk(-P_1 l \cdot (CB - \alpha) - 5 \cdot (\Delta - G)) - 16(pq \cdot (P_1 q \cdot (\alpha + \beta) + 5 \cdot G) + P_1 q \cdot (P_1 l \cdot CB + 5 \cdot \Delta)
\end{aligned}$$

$$\begin{aligned}
& +.5 \cdot P_1 l \cdot \Delta - .5 \cdot N + 16 m_i^2 \cdot p q \cdot (CB - \alpha) - 32 m_i^2 \cdot (\tilde{F} - A_3 - B_3) \} \\
& + \frac{1}{2 l q} \frac{V}{M_K} \{ -8(-l q^2 \cdot CB - (P_1 q \cdot (P_1 l \cdot CB + .5 \cdot \Delta) + .5 \cdot \Delta \cdot P_1 l - .5 N) - .5 m_i^2 \cdot \Delta + .5 N) \\
& + l q \cdot ((P_1 k + P_1 l) \cdot \alpha - B_3) + l k \cdot (P_1 q \cdot \alpha + .5 G) + .5 G \cdot q k \\
& + 2(.5(P_1 k + P_1 l) \cdot (\Delta + m_i^2 \cdot CB) - (m_i^2 + P_1 q) \cdot F - J + (P_1 q \cdot P_1 l + q k \cdot l q) \cdot CB + .5 P_1 l \cdot \Delta) \\
& - 8(-2 l k \cdot l q \cdot \alpha + l k \cdot (P_1 q \cdot \alpha + .5 G) - 2(P_1 k \cdot (P_1 l \cdot \alpha + .5 G) - (P_1 l \cdot (B_3 - P_1 l \cdot \alpha) + .5 B_5 - .5 P_1 l \cdot G)) \\
& + l q \cdot ((P_1 k + P_1 l) \cdot \alpha - B_3) + 2(P_1 l \cdot (B_3 - P_1 l \cdot \alpha) - (P_1 l \cdot (B_3 - P_1 l \cdot \alpha) - .5 B_5 - .5 P_1 l \cdot G)) \\
& + .5 q k \cdot G - (P_1 q \cdot (P_1 l \cdot CB + .5 \Delta) + .5 P_1 l \cdot \Delta - .5 N) + m_i^2 (2 P_1 l \cdot CB - .5 \Delta) + N + l q^2 \cdot CB) \} \\
& - 32 m_i^2 ((P_1 k + P_1 l) \cdot \alpha - B_3) + 16 m_i^2 ((P_1 k + P_1 l) \cdot CB - F) \} \\
(b4) = & \frac{\alpha}{2\pi} |C_2|^2 m_i \{ \\
& p q \cdot \left( \frac{A}{M_K} \right)^2 \{ 32(P_1 p \cdot (\alpha + \beta) - A_3 - B_3 - C_3) - 16(P_1 p \cdot CB - F) - 8(2C_3 - q k \cdot \beta) + 8(P_1 l - P_1 k) \cdot \beta + \\
& 16(P_1 l + P_1 k)(\alpha + \beta - CB) + 16(F - B_3) \\
& - \frac{1}{p q} (-16(P_1 p \cdot (A + B) - A_3 - B_3) + 16 q k \cdot (P_1 p \cdot (\alpha - B_3))) \\
& + \frac{1}{p q} (-8(-p q \cdot C_3 + (P_1 k + P_1 l) \cdot B_3 - I + A_4) + 8 p q (-p q \cdot \beta + (P_1 k + P_1 l) \cdot \alpha - C_3 - F + A_3) \\
& + 8(-p q \cdot q k (CB - \alpha - \beta) + q k (F - B_3 - C_3)) - 8(p q \cdot (P_1 q \cdot CB + .5 \Delta) - (P_1 q \cdot F + .5 J)) \\
& - 8(p q \cdot ((P_1 k + P_1 l) \cdot CB - F) - ((P_1 k + P_1 l) \cdot F + I)) + 8 p l (-p q \cdot (CB - \alpha) + F - A_3 - B_3)) \\
& + 8 M_K^2 q l (CB - \alpha) \} \\
& + p q \left( \frac{A \cdot V}{M_K^2} \right) \{ 16(C_3 - P_1 l \cdot \beta) - 16 q k \cdot \beta - 16(F - P_1 l \cdot CB - B_3 - C_3 + P_1 l \cdot (\alpha + \beta)) \\
& + 16 k l \cdot \beta + 16 m_i^2 \cdot \beta \\
& \frac{1}{p q} (-16(I - B_4 - P_1 p \cdot (F - B_3)) + 8(q k \cdot (F - B_3) + p q (B_3 - P_1 l \cdot \alpha) + .5 p k \cdot \tilde{G}) - 16(-p k \cdot (F - B_3) + p q \cdot C_3) \\
& - 8(q k \cdot p l + q l \cdot p k - p q \cdot k l) \cdot (CB - \alpha) - 16 q l \cdot (q k \cdot \beta + p k \cdot (CB - \alpha))) \} \\
& - \left( \frac{A \cdot V}{M_K^2} \right) \{ 16(P_1 p \cdot P_1 l \cdot \alpha + .5 G \cdot P_1 p - .5 B_5) - 16 q k \cdot (P_1 p \cdot \alpha - B_3) - 32 k \cdot (P_1 p \cdot \alpha - B_3) \\
& - 8(-P_1 k \cdot B_3 + B_4 - p k \cdot l q \cdot \alpha + P_1 p \cdot P_1 l \cdot \alpha + .5 G \cdot P_1 p - .5 B_5) \\
& - 8(-p l \cdot ((P_1 k + P_1 l) \cdot \alpha - B_3) - k l \cdot (P_1 p \cdot \alpha - B_3) - .5 G \cdot p k)
\end{aligned}$$

$$\begin{aligned}
& -8(-.5P_1k \cdot \Delta + lq \cdot qk \cdot CB - P_1p \cdot F + P_1q \cdot P_1l \cdot CB) + 8(-qk((P_1k + P_1l) \cdot \alpha - B_3) - kl \cdot P_1q \cdot \alpha - .5G(kl + qk)))\} \\
& + \left(\frac{V}{M_K}\right)^2 \{-16(-(P_1k + P_1l) \cdot B_3 + B_4 + lq \cdot pk \cdot \alpha) - 4m_l^2(CB - \alpha) - 8pq(F - P_1l \cdot CB - B_3) - 8qk \cdot pq \cdot \beta \\
& + 8(pq \cdot (P_1k - B_3) - pk \cdot (P_1q \cdot \alpha + .5G)) + 16(-pl \cdot ((P_1k + P_1l) \cdot \alpha - B_3) - .5G \cdot pk) \\
& + 4qk \cdot pq \cdot (CB - \alpha - \beta) - 4(pq \cdot ((P_1k + P_1l) \cdot CB - F) - pk(lq \cdot CB + .5\Delta)) \\
& - 4(lq \cdot pk - lk \cdot pq) \cdot (CB - \alpha) + 8ql \cdot pq \cdot \beta - 8(pk \cdot ql + qk \cdot pl) \cdot (CB - \alpha))\}
\end{aligned}$$

## GENERAL CONCLUSIONS

In the high energy or high temperature regime, the standard model shows that anomalous baryon number violation is not suppressed. It is believed to be mediated by the sphaleron configuration which exists due to the non-trivial vacuum structure of the non-Abelian gauge theories even in the weak coupling limit. The energy of the sphaleron sets the energy scale where this non-perturbative phenomenon actually happens. From our investigation, we conclude that the sphaleron energy is a remarkably stable quantity against the new physics, expected at the energy scale  $\Lambda \geq 1$  TeV, through the higher dimension operators (greater than 4). Therefore, the calculation of baryon number violation rate based on the SU(2)-Higgs model is not significantly affected.

In the low energy region, CP violation, one of the necessary conditions to explain baryon number violation, can be manifested in  $K^+ \rightarrow \mu^+ \nu_\mu \gamma$  as the transverse muon polarization  $P_\perp$ . It is also possible to have a non-zero  $P_\perp$  even in the absence of direct CP-violation in this process. This is due to the electromagnetic final state interaction. We calculated this effect and the result shows that the order of magnitude is  $10^{-3}$ . This may be a useful calibration for future experiments.



## ACKNOWLEDGEMENTS

As I sit back and think about this work and the last six years, I am compelled to say that, in my mind, the formal aspects of this experience are overshadowed by the less formal knowledge gained from interactions with those I have met. With this in mind, I would like to take a moment to thank those who unselfishly stood by me with no motivations except friendship and concern.

First and foremost, I like to express my gratitude to my thesis advisor, Professor Bing-Lin Young, for teaching me how to think and how to solve some physics problems. His patience and kindness made it possible for me to finish this work.

I want to thank Dr. Spencer for his enormous efforts to in developing the sphaleron minimization routine. I also want to thank Dr. Chan for his help in fixing problems related to running PAW and debugging my computer code.

A warm thanks to all my fellow Korean students who have spent the last six years with me in Ames. A special thanks to Mr. B. K. Cho and Mr. B. J. Suh for solving computer-related problems .

A special thanks to David Schwellenbach, my first officemate and my mechanic. His support made it possible for me to survive in America for six years. His proof reading of this thesis made it more readable.

I also would like to thank Dr. Geng for his suggestion to work on the second part of this thesis. I will remember those painful days I spent with him struggling to produce the final results.

Thanks are due to innumerable others; if I have not mentioned them it is not because I do not appreciate their help, but because I am tired.

Finally, I would like to thank my wife, Hyun-Jeong, for her endless love and understanding in the last four years. I would like to dedicate this thesis to her. I almost forgot to mention the little devil, my baby, Young-In. She is terrible but she is cute!

This work was performed at Ames Laboratory under Contract No. W-7405-eng-82 with the U.S. Department of Energy. The United States government has assigned the DOE report number IS-T 1684 to this thesis.